Hedonic Price Indexes With Unobserved Product Characteristics, and Application to Personal Computers

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We show that hedonic price indexes may be biased when not all product characteristics are observed. We derive two primary sources of bias. The first source is a classical selection problem that arises due to changes over time in the values of unobserved characteristics. The second comes from changes in the implicit prices of unobserved characteristics. Next we show that the bias can be corrected for under fairly general assumptions using extensions of factor analysis methods. We test our methods empirically using a new comprehensive monthly dataset for desktop personal computer systems. For these data, we find that the standard hedonic index has a slight upward bias of approximately 1.4% per year. We also find that omitting an important characteristic (CPU benchmark) causes a large bias in the index with standard methods, but that this bias is essentially eliminated when the proposed correction is applied.

KEY WORDS: Factor analysis.

1. INTRODUCTION

In recent years, U.S. statistical agencies have dramatically increased their use of hedonic methods in constructing official price indexes. Although the first use of hedonic methods in the consumer price index did not occur until 1987, according to Landefeld and Grimm (2000), approximately 18% of U.S. GDP final expenditures are now deflated using indexes created using hedonic methods, and this number is rapidly growing (see Moulton 2001). Other official indexes, such as the Census Bureau's single-family housing index and the BEA computer price index, used hedonic methods before their adoption in the consumer price index.

Hedonic methods are being introduced into official indexes to correct for two well-known problems with traditional matched-model methods. First, in markets with rapid product turnover, the matched-model index cannot be properly calculated, because it is impossible to measure the prices of new products before they enter and of old products after they exit. Pakes (2003) showed that if the matched-model index is calculated only for those products that remain in the sample, then it is subject to a selection bias, because the products that exit tend to be the ones that are less profitable. Second, the matched-model index does not account for quality change. All price changes, even those associated with improvements in some product characteristics, go into the index.

A long-standing problem with hedonic methods that has been widely recognized (Court 1939; Griliches 1961; Triplett 1969; Griliches and Ohta 1986) but remains unresolved is that typically not all product characteristics are observable by researchers constructing price indexes. The importance of unobserved characteristics has been demonstrated in recent work on demand estimation (e.g., Berry, Levinsohn, and Pakes 1995; Nevo 2001; Bajari and Benkard 2003). Another indication that unobserved characteristics may be important is the fact that it is often the case that hedonic price regressions have a low goodness of fit as measured by the R^2. For example, Pakes (2003) reported R^2's for computers in the range of .26–.52, and Cockburn and Anis (1998) reported R^2's for arthritis drugs in the range of .26–.29. Very low R^2's are not always the case. For example, Berndt, Griliches, and Rappaport (1995) reported R^2's of .77–.83 for computers, and Griliches (1961) reported R^2's in the range of .84–.97 for automobiles.

These observations motivate our three main research questions. First, what explains the errors made in the typical hedonic price regression? Candidate explanations include measurement error in prices, unobserved product characteristics, and approximation error due to functional form. The answer to this question is important, because if price regression errors reflect, for example, only measurement error in prices, then all of the assumptions of standard hedonic methods are satisfied. Second, if the hedonic regression errors reflect unobserved product characteristics, then to what extent is there a bias in the price index? Finally, is it possible to construct hedonic price indexes that fully account for unobserved characteristics?

In Section 2 we show that if some product characteristics are not observed, then hedonic price indexes may be biased, and that this bias comes primarily from two sources. The first source of bias is a classical selection problem that results when the average value of the unobserved characteristics for products in the market changes over time. In ordinary least squares (OLS) estimates, the average value of the unobserved characteristics is absorbed into the period mean of the hedonic regression. This introduces a bias when the estimated hedonic surface from one period is used to predict the prices of products not observed in that period. For example, if the average value of unobserved characteristics is improving over time, then in later periods, hedonic methods would typically overpredict the prices of products that had dropped out of the sample in previous periods. In this example, the price index would exhibit an upward bias.

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The second source of bias is more subtle and results from changes in the implicit prices of the unobserved characteristics over time. Consider the following simple example. Suppose that we wish to calculate a price index between two periods, t and $t+1$. For simplicity, assume that all products are observed in each period, so that both the matched-model index and the hedonic index are defined and there is no selection problem. Assume that there are two observed characteristics, $\mathbf{x}_1$ and $\mathbf{x}_2$ (e.g., CPU speed and RAM), and one unobserved characteristic, $\boldsymbol{\xi}$ (e.g., quality). Suppose that the relationship between prices and product characteristics (both observed and unobserved) is linear, so that in time period t,

$$p_{j,t} = \beta_{0,t} + \beta_{1,t}x_{1,j} + \beta_{2,t}x_{2,j} + \beta_{3,t}\xi_j.$$

Note that the price function is allowed to vary over time, because the coefficients may change between periods.

Suppose that the econometrician is able to consistently estimate the intercept and the coefficients for the observed product characteristics, $\beta_{1,t}$ and $\beta_{2,t}$. Let $p_t(\mathbf{x}_j)$ denote the predicted price of product j at time t using the hedonic surface,

$$p_t(\mathbf{x}_j) = \beta_{0,t} + \beta_{1,t}x_{1,j} + \beta_{2,t}x_{2,j}.$$

In our example it is easy to see that the matched-model index and the hedonic price index differ due to changes in the valuation, $\beta_{3,t}$, of the unobserved characteristic. The matched-model price adjustment between two periods, t and $t+1$, for product j is

$$\begin{aligned} p_{j,t+1} - p_{j,t} = \beta_{0,t+1} - \beta_{0,t} &+ (\beta_{1,t+1} - \beta_{1,t})x_{1,j} \\ &+ (\beta_{2,t+1} - \beta_{2,t})x_{2,j} + (\beta_{3,t+1} - \beta_{3,t})\xi_j, \end{aligned}$$

and the price adjustment using the hedonic surface is

$$\begin{aligned} &p_{t+1}(\mathbf{x}_j) - p_t(\mathbf{x}_j) \\ &\quad = \beta_{0,t+1} - \beta_{0,t} + (\beta_{1,t+1} - \beta_{1,t})x_{1,j} + (\beta_{2,t+1} - \beta_{2,t})x_{2,j}. \end{aligned}$$

In this example the matched-model adjustment is the correct adjustment, and the hedonic adjustment is incorrect. The hedonic adjustment leaves out the term that revalues the unobserved characteristic, $(\beta_{3,t+1} - \beta_{3,t})\xi_j$. Because the aggregate price index is a weighted average of the individual price adjustments, the aggregate price index typically would also be incorrect. Note that in this simple example, if $E[\xi_j|\mathbf{x}_j] = 0$, then the hedonic adjustment is an unbiased estimate of the true adjustment. However, we show in Section 2 that typically there would still be a statistical bias in the price index.

In Section 2 we present our basic model and derive an analytical expression for the bias due to the two sources noted earlier. Table 1 provides an empirical example of the bias obtained in the price index when an important characteristic is known to be omitted. For our data on desktop personal computer (PC) systems (more details of which are provided in Sec. 4.1), the table shows chained Fisher price indexes constructed using a standard hedonic approach (left columns) and then again using the same approach but with the CPU benchmark omitted (right columns). As shown in the table, the indexes with the CPU benchmark omitted exhibit substantial bias. Over just 29 months, the difference in overall inflation is approximately 9%, with the biased indexes showing less deflation. This variation is larger than any variation that we were able to achieve through alternative methods of constructing the index or alternative functional forms; thus we view it as potentially significant.

Table 1. Left-Out Characteristics Bias

	Chained Fisher indexes	
Period	*All characteristics included*	*CPU benchmark omitted*
Aug'97	100.0	100.0
Sep'97	92.3	93.4
Oct'97	85.3	86.8
Nov'97	81.0	83.9
Dec'97	76.0	78.8
Jan'98	66.9	68.9
Feb'98	65.1	68.4
Mar'98	62.0	65.8
Apr'98	59.3	64.3
May'98	54.3	59.1
Jun'98	51.4	56.4
Jul'98	47.8	53.0
Aug'98	43.5	49.5
Sep'98	41.0	47.7
Oct'98	37.4	43.7
Nov'98	35.5	42.3
Dec'98	32.0	38.5
Jan'99	29.7	37.0
Feb'99	28.8	37.1
Mar'99	27.9	37.5
Apr'99	27.2	37.3
May'99	25.4	34.9
Jun'99	22.5	30.4
Jul'99	21.5	30.7
Aug'99	20.1	29.5
Sep'99	18.2	27.6
Oct'99	17.4	27.4
Nov'99	16.6	26.5
Dec'99	15.7	25.1

In Section 3 we show how factor analysis methods can be extended to construct a fairly general statistical test for the presence and even the dimension of the unobserved product characteristics. The intuition for this test is that for products with similar values of the unobserved characteristics, the price regression errors should move similarly over time. Next we show how to use similar methods to construct hedonic indexes that account for the unobserved characteristics. If the unobserved characteristic is one-dimensional, then it is possible to consistently estimate the hedonic surface (including recovering the unobserved characteristics) using a completely general functional form. If the unobserved characteristics are of two or more dimensions, then it is possible to consistently estimate the hedonic surface as long as there is a representation of the surface that is additively separable in the unobserved characteristics. Finally, we show that this methodology works in general if the unobserved characteristics are correlated with each other and, in certain cases, if they are correlated with the observed characteristics.

We apply our methods to a new dataset for desktop PCs. We find in these data that the dimension of the unobserved characteristics is likely to be either two or three. However, we also find that there were insufficient data to enable precise estimates of the price index when the unobserved characteristics were allowed to have dimension greater than one. Given the comprehensiveness of the data, this result sheds doubt on the practical ability to correct price indexes in the multidimensional case. However, these difficulties were exacerbated in our data by the extremely high rate of product turnover and the relatively high measurement error in the price data. Therefore, we believe that correcting for a multidimensional unobservable may be possible in other datasets with less-rapid product turnover and better price measurement. Based on the results of using a one-dimensional unobserved characteristic, we find that the standard hedonic index is upwardly biased by approximately 1.4% per year, and that this bias is due primarily to selection. Specifically, the unobserved characteristics for PCs are improving over time, and this upwardly biases the standard hedonic price index.

We further test our estimation approach by leaving out an important characteristic (the CPU benchmark) and reestimating the price index. Although the standard hedonic index is severely biased in this case, our approach essentially removes the bias even if only a one-dimensional unobserved product characteristic is used (see Table 9 in Sec. 4.2.3). Thus, although the foregoing results suggest that correcting the index for a multidimensional unobservable may be difficult, these results show that corrections based on a one-dimensional unobservable provide a good approximation to the multidimensional case.

Our results suggest that there is a trade-off between the hedonic and matched-model approaches. Hedonic methods are better at capturing quality change and also can solve the product entry and exit problem of the matched-model approach. However, hedonic methods may be biased due to unobserved characteristics. Our approach of including unobserved product characteristics in the hedonic index can be viewed as achieving a middle ground between the two standard approaches. Our approach also lies between the two standard approaches in terms of data requirements. A limitation of our approach is that products must be observed in several time periods, or several spatially separated markets, to estimate the vector of unobserved product characteristics. The number of periods required depends on the dimension of the unobserved characteristics. This falls considerably short of that required to construct the matched-model index, for which every product must be observed in every period.

2. AN EXPRESSION FOR THE BIAS DUE TO UNOBSERVED CHARACTERISTICS IN HEDONIC PRICE FUNCTIONS

2.1 Model and Notation

We assume that the econometrician has data for $t = 0, \dots, T$ periods and, without loss of generality, we assume that the base period of the price index is period $t = 0$. It would not change anything in our analysis to consider instead either T spatially separated markets or a total of T observations for a set of spatially separated markets over time. We assume that each commodity, j, can be represented as a finite-dimensional vector of attributes. In most applications, the economist does not observe all of the product attributes relevant to the consumer. Therefore, in the model the economist perfectly observes the first K attributes, which we denote by the vector $\mathbf{x}_j = (x_{j1}, \dots, x_{jK})$, but does not observe an L-vector of attributes $\boldsymbol{\xi}_j = (\xi_{j1}, \dots, \xi_{jL})$.

Let $\mathcal{C}_t$ be the set of products in market t and denote the set of products that are available in both markets s and t by $\mathcal{C}_{s,t} = \mathcal{C}_s \cap \mathcal{C}_t$. Let F_t be the joint distribution of $(\mathbf{x}, \boldsymbol{\xi})$ in market t with support $\mathcal{X}_t \subset \mathbb{R}^{K+L}$, where $\mathcal{X}_t$ is assumed to be compact.

Implicit in the foregoing notation is the assumption that products are readily identifiable in the sense that it is possible to identify the same product across different time periods t. Under this assumption, the entire vector of product characteristics, $(\mathbf{x}_j, \boldsymbol{\xi}_j)$, is fixed across markets for each product. If a product's characteristics change between two periods, then we define the two products to be different products.

The assumption that a product's characteristics stay fixed over time may be unrealistic in some industries. For example, if one characteristic of a product is the manufacturer's reputation for providing good service, then that could change over time even if the physical aspects of the product do not. Examples of this might include the average hold time on a company's customer service hotline.

2.2 Price Index Formulas

In this article we concentrate on what we believe are the most commonly used forms of the price index: plutocratic weighted average indexes with base period (Laspeyre) or reference period (Paasche) weights. We define the standard matched-model indexes as

$$\mathrm{M}_t^L = \frac{\sum_{j \in \mathcal{C}_0} p_{jt} q_{j0}}{\sum_{j \in \mathcal{C}_0} p_{j0} q_{j0}} \tag{1}$$

and

$$\mathrm{M}_t^P = \frac{\sum_{j \in \mathcal{C}_t} p_{jt} q_{jt}}{\sum_{j \in \mathcal{C}_t} p_{j0} q_{jt}}. \tag{2}$$

Standard results show that M_t^L is an upper bound and M_t^P is a lower bound to the exact price index. These correspond to the classical bounds of Konus (1939) (see also Pakes 2003 for ways of deriving these bounds more generally). In Section 4 we find that, due to a high rate of product turnover, we must instead apply the "chained" versions of these indexes, in which the weights are constantly updated from one period to the next. Therefore, we also calculate chained Fisher indexes, because several authors (e.g., Aizcorbe, Corrado, and Doms 2003) have argued that the chained Fisher index provides a better approximation to the true index in markets with high product turnover.

Hedonic methods substitute prices predicted from the estimated hedonic surface, $p(\mathbf{x})$, into (1) and (2) in the place of actual prices, not all of which are observed. The primary differences between hedonic methods arise in the details of how prices are predicted and whether the predicted prices should always be used or whether they should be used only where actual prices are unavailable, or some combination of these options. In this article we compare the matched-model indexes (M_t^L and M_t^P) with hedonic indexes (H_t^L and H_t^P) in which all of the prices are replaced with prices predicted by the hedonic index,

$$\mathrm{H}_t^L = \frac{\sum_{j \in \mathcal{C}_0} p_t(\mathbf{x}_j) q_{j0}}{\sum_{j \in \mathcal{C}_0} p_0(\mathbf{x}_j) q_{j0}} \tag{3}$$

and

$$\mathrm{H}_t^P = \frac{\sum_{j \in \mathcal{C}_t} p_t(\mathbf{x}_j) q_{jt}}{\sum_{j \in \mathcal{C}_t} p_0(\mathbf{x}_j) q_{jt}}. \tag{4}$$

Our approach differs slightly from the methods proposed by Pakes (2003), which substitutes all prices in the numerator with prices predicted using the hedonic surface but uses actual prices in the denominator. It also differs from the method used by the Bureau of Labor Statistics (BLS), which uses a hybrid of the hedonic and matched-model methods that substitutes predicted prices only in cases where products drop out of the sample. However, our techniques can just as easily be applied to hedonic indexes of those forms. The alternative methods suggested by Pakes (2003) and used by the BLS are in part designed to allow construction of the index under time constraints. We ignore these practical issues in this article.

2.3 An Analytical Expression of the Unobserved Characteristics Bias for the Linear Case

To better understand how unobserved characteristics lead to bias in the aggregate price index, in this section we derive analytical expressions for the bias in the index. Because this is difficult to do in general, we concentrate on the simple case in which the price function is linear. We thus write the price function as

$$p_{jt} = \beta_{0,t} + \mathbf{x}_j'\boldsymbol{\beta}_{\mathbf{x},t} + \boldsymbol{\xi}_j'\boldsymbol{\beta}_{\xi,t}, \tag{5}$$

where both $\mathbf{x}_j$ and $\boldsymbol{\xi}_j$ are vectors.

In characterizing the bias in H^L, it is helpful to rewrite the index as

$$\begin{aligned} \mathrm{H}_t^L &= \frac{\sum_{j\in\mathcal{C}_0} p_t(\mathbf{x}_j)q_{j0}}{\sum_{j\in\mathcal{C}_0} p_0(\mathbf{x}_j)q_{j0}} \\ &= 1 + \frac{\sum_{j\in\mathcal{C}_0}(p_t(\mathbf{x}_j) - p_0(\mathbf{x}_j))q_{j0}}{\sum_{j\in\mathcal{C}_0} p_0(\mathbf{x}_j)q_{j0}}, \end{aligned} \tag{6}$$

where the functions $p_t(\mathbf{x}_j)$ are the hedonic surface in period t, which are only a function of the observed characteristics, as is common in practice.

Consider what happens if we estimate (5) using standard techniques. Suppose, for the sake of simplicity, that we estimate (5) under the assumption that $\boldsymbol{\xi}$ and $\mathbf{x}$ are mean independent, $E_t[\boldsymbol{\xi}|\mathbf{x}] = E_t[\boldsymbol{\xi}] = \boldsymbol{\mu}_t$. If the mean independence assumption holds and there are a large number of observations in each period, then the parameter estimates obtained from the T regressions are

$$\tilde{\beta}_{0,t} \approx \beta_{0,t} + \boldsymbol{\mu}_t'\boldsymbol{\beta}_{\xi,t} \tag{7}$$

and

$$\tilde{\boldsymbol{\beta}}_{\mathbf{x},t} \approx \boldsymbol{\beta}_{\mathbf{x},t}. \tag{8}$$

Note that the intercept captures the average change over time in both the price of $\boldsymbol{\xi}$ and the mean of $\boldsymbol{\xi}$.

We can now use (7) and (8), in conjunction with (6), to characterize the bias in H. The bias in the numerator of H^L is

$$\begin{aligned} &\mathrm{Bias}\big(\mathrm{Num}(\mathrm{H}^L)\big)_t \\ &\quad = (\boldsymbol{\mu}_t - \boldsymbol{\mu}_0)'\boldsymbol{\beta}_{\xi,t}Q_0 + \sum_{j\in\mathcal{C}_0}(\boldsymbol{\mu}_0 - \boldsymbol{\xi}_j)'(\boldsymbol{\beta}_{\xi,t} - \boldsymbol{\beta}_{\xi,0})q_{j0}, \end{aligned} \tag{9}$$

where Q_0 is total sales of the good in the base period. Similarly, the bias in the numerator of H^P is

$$\begin{aligned} &\mathrm{Bias}\big(\mathrm{Num}(\mathrm{H}^P)\big)_t \\ &\quad = (\boldsymbol{\mu}_t - \boldsymbol{\mu}_0)'\boldsymbol{\beta}_{\xi,0}Q_t + \sum_{j\in\mathcal{C}_t}(\boldsymbol{\mu}_t - \boldsymbol{\xi}_j)'(\boldsymbol{\beta}_{\xi,t} - \boldsymbol{\beta}_{\xi,0})q_{jt}, \end{aligned} \tag{10}$$

where Q_t is total sales of the good in the reference period.

The expressions for the bias in the numerator involve two main terms. The first term depends on how much the mean of $\boldsymbol{\xi}$ changes over time and thus reflects selection bias. If there is no selection, such that mean of $\boldsymbol{\xi}$ is constant over time, then the first term is 0. The second term reflects the extent to which the unobserved characteristics are revalued over time, $(\boldsymbol{\beta}_{\xi,t} - \boldsymbol{\beta}_{\xi,0})$. If the value of the unobserved characteristics is constant over time then the second term is 0. These are the two sources of bias mentioned in Section 1. Note also that revaluation of the unobserved characteristics would not bias the index if there were no selection and the quantity-weighted mean of $\boldsymbol{\xi}$ were the same as its unweighted mean, $\boldsymbol{\mu}$. Interestingly, this suggests that if there were no selection problem, then using quantity weights in the hedonic regression would eliminate the unobserved characteristics bias in the index.

The expressions for the bias in the denominator involve similar terms. The bias in the denominator of H^L is

$$\mathrm{Bias}\big(\mathrm{Den}(\mathrm{H}^L)\big)_t = \boldsymbol{\beta}_{\xi,0}\sum_{j\in\mathcal{C}_0}(\boldsymbol{\mu}_0 - \boldsymbol{\xi}_j)'q_{j0}, \tag{11}$$

whereas the bias in the denominator of H^P is

$$\mathrm{Bias}\big(\mathrm{Den}(\mathrm{H}^P)\big)_t = \boldsymbol{\beta}_{\xi,t}\sum_{j\in\mathcal{C}_t}(\boldsymbol{\mu}_t - \boldsymbol{\xi}_j)'q_{jt}. \tag{12}$$

The bias in the denominator of the index reflects the extent to which the quantity-weighted mean of the unobserved characteristic differs from its unweighted mean. It is difficult to sign this bias in general, because the quantity weights depend on consumer tastes. Assuming that the unobserved characteristics carry positive prices, if demand is higher for goods with higher values of the unobserved characteristics, then the denominator is downwardly biased, leading to an upward bias in the index. Note also that the bias is constant over time for the Laspeyre index and is likely to be fairly constant for the Paasche index. This means that if the denominator is biased downward, then price changes in the index for all periods will be biased upward. Based on our experiences, our prior is that this source of bias is likely to be less important than the previous two.

The bias in the index as a whole is easiest to evaluate asymptotically because, by the Slutzky theorem, the bias in the index can then be evaluated by considering the biases in the numerator and denominator separately. This leaves three overall sources of bias, two in the numerator (selection and repricing of the unobserved characteristic) and one in the denominator (the difference between the quantity weighted mean of $\boldsymbol{\xi}$ and its unweighted mean). The bias in the index will reflect the sum of these three sources.

In our opinion, there are many industries in which the mean of the unobserved characteristics and the price of the unobserved characteristics are likely to change over time, particularly high-technology industries. In that case, it is likely that there would be unobserved characteristic bias in standard hedonic indexes.

3. MODELING UNOBSERVABLES IN THE HEDONIC PRICE FUNCTION

In this section we outline an approach to estimating hedonic price functions in the presence of unobserved characteristics. Our approach is similar to the factor analysis literature, especially the work of Lawley and Maxwell (1971), Goldberger (1974), and Cragg and Donald (1995, 1997), except that we have found it necessary to extend that work in several ways, most notably to account for selection.

3.1 The Hedonic Price Function

Bajari and Benkard (2003) provided a set of primitive conditions under which there exists a price surface, denoted by $p_t(\mathbf{x}_j, \boldsymbol{\xi}_j)$, in each market t. For the remainder of the article we implicitly rely on the results of their theorem in the sense that we assume that there exists a function mapping product characteristics to prices. Bajari and Benkard (2003) treat $\boldsymbol{\xi}_j$ as one-dimensional; however, extending the theorem to the case in which $\boldsymbol{\xi}_j$ is L-dimensional is straightforward. If the assumptions of the theorem were to not hold, then the hedonic approach could still be viewed as an approximation to the truth. But we cannot say how good the approximation would be without making additional assumptions.

To simplify the analysis and estimation, we assume that the price function can be written as additively separable in the observed and unobserved product characteristics and linear in the unobserved characteristics,

$$p_{jt} = f_t(\mathbf{x}_j) + \boldsymbol{\beta}'_{\xi,t}\boldsymbol{\xi}_j + \nu_{jt}, \tag{13}$$

where $f_t(\cdot)$ is a function, possibly of unknown parametric form, and ν_{jt} represents measurement error in the observed price.

Equation (13) places some restrictions on the functional form of the price function, but retains perhaps more generality than is first apparent. All of the analysis that follows is general to nonlinear transformations of the right-side and left-side variables. In addition, $f_t(\cdot)$ can be a general nonparametric function within any one of those forms. Because we allow the unobserved product characteristics to be correlated with one another, higher-order terms in ξ may appear as additional dimensions. Because our analysis is general to the case where $\boldsymbol{\xi}$ is correlated with $\mathbf{x}$ (if the relationship is stable over time; see Sec. 3.3), interactions between $\boldsymbol{\xi}$ and $\mathbf{x}$ may also appear as additional dimensions. We allow for measurement error in prices, because in our experience with price data in I.O. applications, we have found that this can happen for various reasons, and furthermore, we believe it to be true in our data.

For ease of exposition, in this section we maintain several assumptions that we later relax. First, we assume that the unobserved product characteristics are mean independent of the observed product characteristics. This assumption is common in the hedonics literature implicitly and also in the literature on demand estimation explicitly. It seems likely that it is violated to some extent in practice, so we show that it is possible to substantially relax this assumption in Section 3.3. Second, we assume that the measurement error is iid and independent of $\mathbf{x}$ and $\boldsymbol{\xi}$. It is straightforward to generalize the specification of the measurement error in several ways, including AR(p), and heteroscedasticity of unknown form. We consider the latter case here. Finally, the analysis is substantially easier to follow if we assume that there is no selection in the data; that is, we assume that the distribution of $\boldsymbol{\xi}_j$ is constant over time. We add selection to the model in Section 3.2.

3.1.1 Estimating $f_t(\cdot)$. Let $\epsilon_{jt} \equiv \boldsymbol{\beta}'_{\xi,t}\boldsymbol{\xi}_j + \nu_{jt}$ represent the error terms in the period-by-period hedonic price regressions. Under the assumptions listed earlier, $E_t[\epsilon_{jt}|\mathbf{x}] = \boldsymbol{\beta}'_{\xi,t}\boldsymbol{\mu}$, where $\boldsymbol{\mu} = E[\boldsymbol{\xi}_j|\mathbf{x}_j]$ is a constant. Therefore, the functions $f_t(\cdot)$ can be estimated using standard techniques. For example, if the parametric form of the functions $f_t(\cdot)$ were known, then they could be estimated using least squares. Otherwise, kernel or series based nonparametric regression techniques could be used. Note that the $f_t(\cdot)$ functions absorb the mean of the unobservable, $\boldsymbol{\beta}'_{\xi,t}\boldsymbol{\mu}$, so at this point the functions can be estimated only up to an additive constant term. (If there is selection, then each function $f_t(\cdot)$ absorbs the period mean of the unobservable, $\boldsymbol{\beta}'_{\xi,t}\boldsymbol{\mu}_t$.) Because these estimation approaches are standard, we omit a detailed discussion of them and proceed as if $f_t(\cdot)$ were known.

3.1.2 Estimating $\boldsymbol{\beta}_{\xi,t}$. What makes it possible to identify and estimate the complete model (13) is the fact that this model places tight restrictions on the covariance matrix of the errors in the hedonic regressions, ϵ_{jt}. To derive those restrictions, we first need to make some normalizations. The normalizations we use are standard to factor analysis and are without loss of generality (see Lawley and Maxwell 1971 for a good discussion). We remind the readers that at this point we are maintaining the assumption that there is no selection. In the event that there is selection, we have to be careful in applying the normalizations; see Section 3.2 for details. We normalize the mean of $\boldsymbol{\xi}$ to be $\mathbf{0}$, $E[\boldsymbol{\xi}_j] = \mathbf{0}$. We also normalize $\boldsymbol{\xi}$ to have covariance matrix $\mathbf{I}_L$ across all periods $0, \dots, T$. The reason that the normalizations are necessary is that $\boldsymbol{\xi}$ is not observable and thus has no inherent units. It is multiplied by a coefficient vector that is also unknown. Thus neither the mean nor the variance of $\boldsymbol{\xi}$ is identified separately from the coefficients $\boldsymbol{\beta}_{\xi}$. Importantly, knowledge of the normalized coefficients is sufficient for construction of the price index.

Let $\boldsymbol{\epsilon}_j$ be the T-vector of errors for product j. Then under the assumptions given earlier,

$$\boldsymbol{\Sigma} \equiv E[\boldsymbol{\epsilon}_j\boldsymbol{\epsilon}'_j] = \boldsymbol{\beta}_{\xi}\boldsymbol{\beta}'_{\xi} + \sigma^2_{\nu}\mathbf{I}_T. \tag{14}$$

Without any restrictions, $E[\boldsymbol{\epsilon}_j\boldsymbol{\epsilon}'_j]$ has $\frac{T(T+1)}{2}$ unique elements. However, our model contains only $T * L + 1$ parameters. Thus for small values of L, the model places significant restrictions on this matrix. In fact, it is possible to estimate the entire matrix of parameters $\boldsymbol{\beta}_{\xi}$ as long as $L \leq \frac{T}{2}$ (approximately). Because most price index applications have data for a large number of time periods or spatially separated markets, the model is typically overidentified for reasonable values of L.

Estimation of $\boldsymbol{\beta}_{\xi}$ can be achieved in several ways. The traditional approach of the factor analysis literature (e.g., Lawley and Maxwell 1971) has been to assume normality for the unobserved product characteristics and then use maximum likelihood. However, such an approach would be inappropriate here, because the model provides us with only first- and second-moment information and nothing more. If we were to assume normality of the unobserved product characteristics, then, in conjunction with the normalization of their covariance matrix to the $\mathbf{I}_L$ matrix, we would be implicitly assuming full independence of the unobserved product characteristics. But we do not want to assume full independence, because we want to allow for functional form flexibility in (13). Thus we instead proceed using generalized method of moments (GMM), with the moment conditions provided by (14). In a previous version of the article we used likelihood methods and found that they led to an overestimate of the number of unobserved product characteristics, L.

Assuming that there is no selection, the model can be estimated as follows. Let

$$\mathbf{S} = \frac{1}{J}\sum_{j=1}^{J} \boldsymbol{\epsilon}_j \boldsymbol{\epsilon}_j'. \tag{15}$$

Because (15) is the empirical counterpart to (14), our model gives us $\frac{T(T+1)}{2}$ unique moment conditions,

$$E[\mathbf{S}] = \boldsymbol{\Sigma}. \tag{16}$$

The natural GMM estimator would minimize a quadratic form in these moment conditions,

$$\{\hat{\boldsymbol{\beta}}_\xi, \hat{\sigma}_\nu\} = \arg\min(\operatorname{vech}\mathbf{S} - \operatorname{vech}\boldsymbol{\Sigma})'\mathbf{A}(\operatorname{vech}\mathbf{S} - \operatorname{vech}\boldsymbol{\Sigma}),$$

for some positive-definite weight matrix $\mathbf{A}$. Under standard conditions, $\hat{\boldsymbol{\beta}}_\xi$ and $\hat{\sigma}_\nu$ are consistent and asymptotically normal for any positive-definite weight matrix $\mathbf{A}$. For example, the $\mathbf{I}$ matrix could be used. Also, under standard conditions,

$$\sqrt{J}(\operatorname{vech}\mathbf{S} - \operatorname{vech}\boldsymbol{\Sigma}) \to \mathrm{N}(\mathbf{0}, \mathbf{V}),$$

and it is well known that the optimal weight matrix to use in the GMM objective function is $\mathbf{A} = \mathbf{V}^{-1}$ (see Hansen 1982). In our application, the number of observations will typically vary by cell of $\mathbf{S}$, so the asymptotic approximations have to be corrected appropriately.

3.1.3 Hypothesis Tests for the Dimension L. The foregoing estimation algorithm is conditional on knowing the dimension L. Cragg and Donald (1997) showed that if the optimal weight matrix is used, then the value of the objective function can also be used as a statistical test for the true dimension of the model. The difficulty in applying the approach of Cragg and Donald (1997) in our application comes in estimating $\mathbf{V}$. Typically, a consistent estimator of $\mathbf{V}$ can be obtained using the sample moments of $\boldsymbol{\epsilon}_j$. For example, an estimator for the covariance between the (q, r) and (s, t) elements of $\mathbf{S}$ is given by

$$\frac{1}{N}\sum_{j=1}^{N}(\epsilon_{j,q}\epsilon_{j,r}\epsilon_{j,s}\epsilon_{j,t} - S_{q,r}S_{s,t}).$$

However, in our application, products tend to not last longer than about 12 months, so there are many combinations of (q, r, s, t) for which there are very few or even no observations. Thus, although it is still possible to estimate $\mathbf{V}$, it is not possible to estimate it very well, in our experience not well enough to construct reliable hypothesis tests. One problem that we had was that the differing number of observations in every cell led to an estimate of $\mathbf{V}$ that was not positive definite due to sampling error and thus was not invertible to obtain the weight matrix. We solve this problem by using subsamples of data for which $\mathbf{V}$ can be estimated well.

A potential problem with these hypothesis tests is that the errors used to calculate the moment conditions in (16) are estimated and thus are not equal to the true error terms. Although this does not affect consistency of the index, the additional noise might influence the hypothesis tests toward causing false rejections (i.e., toward supporting too many unobserved factors). The extent of the problem would likely depend on the number of first-stage observations and on the variance of the measurement error in price. One possible solution to this problem would be to estimate the first and second stages jointly using GMM. (We thank an anonymous referee for suggesting this solution.) That is, the first stage consists of a set of OLS moments (one set for each time period) given by

$$E[\epsilon_{jt}|\mathbf{x}] = 0.$$

These moments could be combined with those in (16) into one large joint GMM estimation procedure. Hypothesis tests based on the joint GMM objective function would then account for first-stage estimation error. The problem with the joint approach is that it has a massive data requirement. Each time period in the first-stage estimation adds a set of K moment conditions, leading to a total of $\mathrm{NMOM} = T * (K + (T + 1)/2)$ moments. To run hypothesis tests, it is necessary to obtain a good estimate of the variance covariance matrix of the moment conditions, which has $\mathrm{NMOM} * (\mathrm{NMOM} + 1)/2$ unique elements. For many datasets, including the one used in this article, this will not be possible. We discuss this issue further in Section 4.

3.1.4 Estimating $\boldsymbol{\xi}$. The foregoing two-step approach provides estimates of all of the model parameters. However, to construct price indexes, it is also necessary to estimate the vector of unobserved product characteristics for each product. The vector of errors for each product j can be written as

$$\epsilon_{jt} = \boldsymbol{\beta}_{\xi,t}'\boldsymbol{\xi}_j + \nu_{jt}. \tag{17}$$

At this point we assume that the parameters $\boldsymbol{\beta}_{\xi,t}'$ are known, because they have been estimated previously.

Because $\boldsymbol{\beta}_{\xi,t}$ is known and the measurement error is iid and independent of everything, (17) becomes a standard linear regression model with $\boldsymbol{\beta}_{\xi,t}$ as the observed covariates and $\boldsymbol{\xi}_j$ as the unknown parameter vector. Estimation of this equation is straightforward via OLS. A problem likely to be encountered is that (17) can be estimated only for those products with prices observed in $T_j \geq L$ periods and, depending on the variance of the measurement error, can be estimated well only if T_j is large. In that case, if $L > 1$, then in general it is not possible to estimate $\boldsymbol{\xi}_j$ for all products, and it may be difficult to estimate $\boldsymbol{\xi}_j$ well unless T_j is large or the variance of the measurement error is small. This also introduces some selection into the index, because some products would have to be dropped in calculating the index.

In application, $\boldsymbol{\beta}_{\xi,t}$ is not known but is instead replaced by a consistent estimator, $\hat{\boldsymbol{\beta}}_{\xi,t}$. This introduces finite-sample bias into the estimates of $\boldsymbol{\xi}$ similar to that of measurement error in the standard regression model. Because $\hat{\boldsymbol{\beta}}_{\xi,t}$ is consistent as the number of products goes to infinity, this bias goes to 0 with the number of products. Provided that there is measurement error ($\sigma_\nu^2 > 0$), consistency of $\hat{\boldsymbol{\xi}}_j$ also requires that the number of time periods (or spatially separated markets) become large for each product. Consistency of $\hat{\boldsymbol{\xi}}_j$ thus would be obtained as both the number of products and the number of time periods become large.

3.2 Selection

An important problem with proceeding using the GMM approach described earlier is that there is substantial selection in our data for PCs. As technology improves, lower-quality products exit, and higher-quality products enter. Thus, for example, it is unlikely that the products that we observe in period 1 are a random sample of products from the distribution of all products observed in all periods, as is required by the moment conditions in (16). In this section we allow there to be selection on both observed product characteristics, $\mathbf{x}_j$, and unobserved product characteristics, $\boldsymbol{\xi}_j$. We continue to assume that the measurement error in price is iid and thus not subject to selection.

Selection introduces two main problems to the analysis. The first problem is that even if the mean of the unobserved product characteristics is normalized to 0 overall, the mean of the unobserved product characteristics is not necessarily 0 among products observed in any given period, $\boldsymbol{\mu}_t \equiv E_t[\boldsymbol{\xi}_j|\mathbf{x}_j] \neq \mathbf{0}$. The same is true among products observed in any pair of periods s and t. One way in which this shows up is that the errors from the hedonic price regressions, $\epsilon_{j,t}$, will include a term in the period mean of the unobserved characteristics,

$$\epsilon_{j,t} = \boldsymbol{\beta}'_{\xi,t}(\boldsymbol{\xi}_j - \boldsymbol{\mu}_t) + \nu_{jt}.$$

This extra term must be accounted for when estimating $\boldsymbol{\xi}_j$. This can be done using multivariate and partitioned regression techniques or an equivalent iterative procedure.

The second problem caused by selection is that we observe the covariance of the errors in the price regression between two periods only for products observed in both periods. If selection influences these covariances, then it is impossible to calculate sample moments that correspond to the population moments given by (16). Formally, for any pair of periods $s, t \in 0, \dots, T$, the moments in (16) represent $E[\epsilon_{j,s}\epsilon_{j,t}]$. Instead, we observe the sample counterpart to the population moment, $E[\epsilon_{j,s}\epsilon_{j,t}| j \in \mathcal{C}_{s,t}]$, where $\mathcal{C}_{s,t}$ represents the set of products observed in periods s and t. Therefore, if we were to proceed as described earlier and ignore the selection problem, then we may bias the estimates of $\boldsymbol{\beta}_\xi$ as well as the statistical-dimension tests.

Our approach to handling selection is two-fold. First, when running the hypothesis tests for the dimension L on subsamples as described earlier, instead of using all data points for all products observed at any point during the subsample, we reduce the data to a balanced panel. Formally, we choose a balanced panel, $\mathcal{C}_{s,\dots,t}$, representing all products observed in all periods, $s, \dots, t$. When running the statistical-dimension tests, we then use the moment conditions in (16), but only for products in the balanced panel,

$$E[\mathbf{S}| j \in \mathcal{C}_{s,\dots,t}] = \boldsymbol{\Sigma}.$$

Note that the fact that we are using a balanced panel means that we are using a selected group of products. For example, because these products were observed over the entire panel, they are likely to be better than products that dropped out at some point. The way in which this selection would show up is that the unobserved characteristics in the balanced panel would have a different (perhaps higher) mean and different covariance matrix than an unselected sample. However, the mean and covariance matrix are normalized away in the estimation, so the fact that they are different than what would be obtained without selection does not matter. What matters for the estimation is that the mean and covariance matrix are held constant across the moment conditions (the entire matrix $\mathbf{S}$). Holding the selection constant over the panel allows us to discern common movements in the price regression errors, which allows us to estimate the coefficients, $\boldsymbol{\beta}_\xi$. As long as there are enough products in the balanced panel to identify the coefficients, the selection no longer matters. The downside of the balanced-panel approach is that it forces us to throw out some of the information available in the data; the upside is that it allows us to test for the dimension L while allowing for selection without restrictions.

The balanced-panel approach does not allow us to estimate the price index as a whole, because very few (if any) products are observed in every period in the data. However, we can extend the intuition of the balanced panel forward in several ways. Perhaps the easiest approach would be to chain together balanced panels for several subsamples of the data in order to construct the whole index. This approach should in principle work, but at the expense of not using all of the available information in the data. Instead, we propose using the moment conditions from overlapping balanced panels in conjunction with each other in order to estimate the overall index. However, in order to do this, we have to explicitly account for the varying selection across different panels.

Suppose that we apply the factor analysis normalizations relative to the products in some group $\mathcal{C}_\eta$, such that the $\boldsymbol{\xi}$'s for those products have mean $\mathbf{0}$ and covariance matrix $\mathbf{I}_L$. For example, the group $\mathcal{C}_\eta$ could be the balanced panel of all products observed in all periods $1, \dots, 10$. Then, as above, these products provide us with a series of moment conditions,

$$E[\mathbf{S}|\mathcal{C}_{1,\dots,10}] = \boldsymbol{\Sigma}_\eta \equiv \boldsymbol{\beta}_\xi \boldsymbol{\beta}'_\xi + \sigma_\nu^2 \mathbf{I}.$$

Now, consider a second group of products, $\mathcal{C}_\tau$. For example, $\mathcal{C}_\tau$ could be the balanced panel of all products observed in all periods $2, \dots, 11$. If we allow the selection process to be completely unrestricted, then we know nothing about the mean and variance of $\boldsymbol{\xi}$ among this second group relative to the normalization from the first group. However, we still have an equivalent set of moment conditions,

$$E[\mathbf{S}|\mathcal{C}_{2,\dots,11}] = \boldsymbol{\Sigma}_\tau \equiv \boldsymbol{\beta}_\xi \boldsymbol{\Psi}_\tau \boldsymbol{\beta}'_\xi + \sigma_\nu^2 \mathbf{I},$$

where $\boldsymbol{\Psi}_\tau$ is the covariance matrix of $\boldsymbol{\xi}_j$ among products in $\mathcal{C}_\tau$. Because of the presence of the new parameters, $\boldsymbol{\Psi}_\tau$, this second set of moment conditions does not provide as much information as the first. However, for small dimensions L, they should still provide a great deal of information. For example, if $L = 1$ then these moment conditions are simply shifted by a constant relative to the first group. In this manner we can use many moment conditions from successive overlapping balanced panels to estimate the parameters $\boldsymbol{\beta}_\xi$ while allowing for selection without restriction. Note that this procedure introduces new incidental parameters, $\boldsymbol{\Psi}_\tau$, for each set of moment conditions used, and therefore increases the computational burden of the estimation.

3.3 The Nonindependent Case

We have already shown that if the unobserved product characteristics are correlated with each other but independent of the observed characteristics, then we can estimate the price index by normalizing them to be uncorrelated with each other. In this section we consider the case where the unobserved product characteristics are also correlated with the observed product characteristics.

We consider the case where the functions $f_t(\cdot)$ are estimated using a nonparametric series estimator. This approach was suggested by Pakes (2003) and is also used in the empirical section (Sec. 4) of this article. It also nests many parametric approaches, including linear, semilog, and log–log, and can be viewed as an approximation to other nonparametric approaches. In that case, the price equation can be written as

$$p_{jt} = \beta_{0,t} + \boldsymbol{\beta}'_{\mathbf{x},t}\boldsymbol{\phi}(\mathbf{x}_j) + \boldsymbol{\beta}'_{\xi,t}\boldsymbol{\xi}_j + \nu_{jt}, \tag{18}$$

where $\boldsymbol{\phi}(\mathbf{x}_j)$ is a $M \times 1$ vector of basis functions of $\mathbf{x}_j$ and $\boldsymbol{\beta}_{\mathbf{x},t}$ is a $M \times 1$ vector of parameters.

Without loss of generality, $\boldsymbol{\xi}_j$ can be written as,

$$\boldsymbol{\xi}_j = \boldsymbol{\mu}_t + \boldsymbol{\gamma}_t\boldsymbol{\phi}(\mathbf{x}_j) + \boldsymbol{\zeta}_{jt}, \tag{19}$$

where $E_t[\boldsymbol{\zeta}_{jt}|\boldsymbol{\phi}(\mathbf{x}_j)] = \mathbf{0}$ and $\boldsymbol{\gamma}_t$ is a $L \times M$ matrix of parameters. The expression (19) represents the period t projection of $\boldsymbol{\xi}_j$ on $\boldsymbol{\phi}(\mathbf{x}_j)$ with respect to the period t sampling distribution of $(\mathbf{x}_j, \boldsymbol{\xi}_j)$. [For clarification, $\boldsymbol{\mu}_t$ and $\boldsymbol{\gamma}_t$ are the coefficients from a regression of $\boldsymbol{\xi}_j$ on $\boldsymbol{\phi}(\mathbf{x}_j)$ for all products observed in period t.] In general, the relationship between $\boldsymbol{\xi}_j$ and $\boldsymbol{\phi}(\mathbf{x}_j)$ may vary over time, depending on such factors as changes in production technology and selection. But suppose that the projection were stable over time, such that

$$\boldsymbol{\xi}_j = \boldsymbol{\gamma}\boldsymbol{\phi}(\mathbf{x}_j) + \boldsymbol{\zeta}_j, \tag{20}$$

where $E_t[\boldsymbol{\zeta}_j|\boldsymbol{\phi}(\mathbf{x}_j)] = \boldsymbol{\mu}_t$ for all time periods, t. Then combining (18) and (20) gives

$$p_{jt} = \beta_{0,t} + (\boldsymbol{\beta}'_{\mathbf{x},t} + \boldsymbol{\beta}'_{\xi,t}\gamma)\boldsymbol{\phi}(\mathbf{x}_j) + \boldsymbol{\beta}'_{\xi,t}\boldsymbol{\zeta}_j + \nu_{jt}. \tag{21}$$

This model is analogous to that estimated earlier for the independent case. Thus, using the approach outlined earlier, we can consistently estimate the quantities $\beta_{0,t}$, $(\boldsymbol{\beta}'_{\mathbf{x},t} + \boldsymbol{\beta}'_{\xi,t}\gamma)$, $\boldsymbol{\beta}_{\xi,t}$, and $\boldsymbol{\zeta}_j$, under the correct assumption that $E_t[\boldsymbol{\zeta}_j|\boldsymbol{\phi}(\mathbf{x}_j)] = \boldsymbol{\mu}_t$. The quantities $\boldsymbol{\beta}_{\mathbf{x},t}$, $\boldsymbol{\gamma}$, and $\boldsymbol{\xi}_j$ are not separately identified/estimable using this approach. However, we do not need these quantities to evaluate $p_t(\mathbf{x}_j, \boldsymbol{\xi}_j)$. The estimable quantities are sufficient for evaluating this function and thus are sufficient for construction of the hedonic price index. Thus, as long as the relationship between $\boldsymbol{\xi}_j$ and $\mathbf{x}_j$ is stable over time, the estimation approach described in the previous section provides consistent estimates of the price index.

Although it is substantially more general than assuming that they are independent, the assumption that the relationship between $\boldsymbol{\xi}_j$ and $\mathbf{x}_j$ is stable over time is somewhat restrictive, particularly if we were considering long panels. However, it would also be possible to use balanced panels in the first-stage hedonic regressions to solve the correlation problem more generally. The reason that balanced panels would work is the same as before. They hold the set of products fixed over time, thus making the relationship between $\boldsymbol{\xi}_j$ and $\mathbf{x}_j$ fixed over time.

4. EMPIRICAL RESULTS

4.1 Data

Our data come from the *PC Data Retail Hardware Monthly Report* and include quantity sold, average sales price, and a long list of machine characteristics for desktop computers sold over a 29-month period from August 1997 to December 1999. This dataset reportedly covers approximately 75% of U.S. retail computer sales. The price data are collected from cash register receipts, and the dataset is constructed by taking total sales of each product over a month and dividing by quantity sold. The dataset thus represents the average retail sales price of the machine in that month. In working with the raw data, we discovered two problems that we felt we needed to address. First, the data for machines with very few sales was highly variable from month to month. Second, sometimes machines are recorded as having been sold at very low prices (e.g., \$.01) when they were in fact taken off the books for other reasons, such as because the unit was stolen. Thus, to remove both of these problems, we dropped all price observations for units that sold fewer than 10 units in a given period. After dropping these observations, 3,853 machines remained from an original sample of approximately 8,000.

The characteristics data included 65 product characteristics, including 23 processor-type dummies and 9 operating system-type dummies. To reduce the dimension of the characteristics space, rather than use the 23 processor-type dummies and the speed rating of the chip as separate characteristics, we instead obtained CPU benchmarks for each machine from *The CPU Scorecard* (*www.cpuscorecard.com*). Despite having considerable variation, a regression of the CPU benchmark variable on processor dummies interacted with chip speed had an R^2 of .995, justifying its use.

Of the remaining 41 characteristic fields, we eliminated those fields that either were not reliable (not always recorded) or applied only to a handful of machines. Despite the need to drop several of the characteristics fields, we were left with an extremely rich set of characteristics. The final characteristics set included nine operating system dummies (Win 3.11, Win 3.1, NT 3.51, NT3.2, NT 4.0, NT, Win 98, Win 95, other) plus CPU benchmark, MMX, RAM capacity, hard drive capacity, SCSI, CDROM, DVD, modem, modem speed, NIC, monitor dummy, monitor size, zip drive, desktop (versus tower), refurbished, dual hard drive, and dual processor, for a total of 26 characteristics.

Tables 2 and 3 contain summary statistics for the final dataset. Table 2 shows that there are approximately 600 machines per month in the data, representing an average of approximately 300,000 units sold. The sales-weighted average price of machines drops by approximately 40% over the 29-month period. The unweighted average price is generally higher, but moves similarly. At the same time, Table 3 shows that sales-weighted average CPU benchmark and sales-weighted average hard drive capacity both rise by approximately a factor of four, whereas sales-weighted average RAM rises by approximately a factor of three. To summarize, prices for a constant-quality machine are dropping rapidly, but consumers are also rapidly sub-

Table 2. Summary of PC Data

Period	Unique machines	Total sales	Average price (unweighted)	Average price (sales-weighted)
Aug'97	577	226,029	1,396	1,422
Sep'97	556	239,417	1,408	1,437
Oct'97	562	211,610	1,411	1,423
Nov'97	517	265,070	1,358	1,351
Dec'97	524	345,153	1,308	1,321
Jan'98	572	328,028	1,224	1,200
Feb'98	525	331,262	1,172	1,217
Mar'98	614	371,337	1,187	1,194
Apr'98	601	260,173	1,206	1,179
May'98	547	210,834	1,182	1,134
Jun'98	660	278,002	1,160	1,111
Jul'98	563	250,110	1,156	1,133
Aug'98	615	345,183	1,177	1,092
Sep'98	649	393,909	1,131	1,113
Oct'98	647	296,737	1,128	1,032
Nov'98	563	428,776	1,046	1,099
Dec'98	644	592,138	1,042	995
Jan'99	593	406,644	981	1,028
Feb'99	569	371,586	998	1,056
Mar'99	675	452,156	1,025	1,046
Apr'99	635	313,716	977	1,061
May'99	608	285,353	968	1,033
Jun'99	692	378,476	947	1,002
Jul'99	614	330,798	878	1,020
Aug'99	616	478,200	841	992
Sep'99	672	571,820	848	953
Oct'99	710	379,487	866	914
Nov'99	661	484,269	861	925
Dec'99	747	664,983	912	879

stituting toward higher-quality machines. The net result is that average purchase prices still drop by 50% over the 29-month period.

Table 3 shows that, despite the fact that our data cover only 29 months, there is considerable shift in the boundaries of the characteristics space over time. The shift in the minimum set of characteristics available is only slight. However, there is a considerable shift upward in the maximum characteristics available, particularly with respect to CPU benchmark and hard drive capacity. Table 3 also leaves out some shifts in the characteristics space with respect to the other product characteristics; for example, in our data, Windows NT 3.51 is unavailable after May 1998.

4.2 Price Index Calculations

4.2.1 Standard Indexes. Table 4 lists matched-model price indexes calculated using the final dataset. Even though our data is quite high-frequency relative to that used by the BLS, the standard matched-model indexes are quite unreliable here, because there is so much attrition in the sample. The standard indexes suffer from a selection bias, present even in the initial periods, as well as from considerable noise in later periods due to there being so few matched products (note the drop from November 1999 to December 1999). In our opinion, Table 4 shows how difficult it is to construct a matched-model price index for a fast-paced high-technology industry like PCs. Even in such a short span as 2 years and even using a very comprehensive dataset covering nearly 4,000 machines, it is nearly impossible to use the matched-model method to construct a reliable price index. On the other hand, there are sufficient observations common to any two neighboring periods so that the chained indexes do not suffer from the same sampling noise problem. However,

Table 3. Summary of Product Characteristics

	CPU benchmark				RAM				Hard drive capacity			
Period	Average	SD	Min	Max	Average	SD	Min	Max	Average	SD	Min	Max
Aug'97	333	178	17	781	24	12	4	128	2,340	1,147	420	7,000
Sep'97	343	194	17	855	25	14	4	128	2,509	1,219	420	7,000
Oct'97	383	203	17	855	27	14	4	128	2,733	1,292	420	7,000
Nov'97	400	216	17	982	26	12	4	128	2,817	1,355	420	7,000
Dec'97	422	213	17	982	27	12	8	128	2,910	1,382	420	8,000
Jan'98	428	219	17	982	27	12	4	128	2,998	1,538	250	12,000
Feb'98	472	222	17	982	30	14	4	128	3,174	1,587	420	12,000
Mar'98	501	226	17	1,130	31	15	8	128	3,302	1,718	420	12,000
Apr'98	532	236	17	1,131	32	15	4	128	3,474	1,826	80	12,000
May'98	572	237	17	1,131	33	15	8	128	3,665	1,832	500	12,000
Jun'98	599	251	17	1,131	36	18	8	128	3,900	2,004	420	12,000
Jul'98	661	252	17	1,131	39	19	8	128	4,239	2,065	800	12,000
Aug'98	700	248	17	1,344	41	21	8	128	4,464	2,148	500	12,000
Sep'98	730	256	17	1,240	44	23	8	128	4,697	2,364	420	16,800
Oct'98	743	271	17	1,240	45	24	8	128	4,794	2,407	540	16,800
Nov'98	802	261	17	1,240	49	27	8	256	5,127	2,481	850	16,800
Dec'98	806	265	17	1,270	51	30	8	256	5,292	2,644	250	18,000
Jan'99	843	264	17	1,468	53	30	8	256	5,490	2,696	250	19,000
Feb'99	899	249	17	1,651	57	32	8	256	5,919	2,982	250	19,000
Mar'99	929	285	17	1,651	57	30	8	256	6,058	3,105	250	20,000
Apr'99	991	275	17	1,651	60	29	8	256	6,449	3,307	250	32,000
May'99	1,049	276	17	1,780	63	30	16	256	6,925	3,337	250	20,400
Jun'99	1,080	303	17	1,814	64	31	4	256	7,221	3,651	340	32,000
Jul'99	1,151	287	17	1,930	68	37	16	512	7,608	3,789	500	32,000
Aug'99	1,183	299	17	2,254	69	34	16	256	7,765	3,829	500	27,000
Sep'99	1,237	328	17	2,347	72	35	16	256	8,202	4,197	500	27,000
Oct'99	1,278	343	17	2,399	72	33	16	256	8,545	4,362	64	27,000
Nov'99	1,329	343	17	2,510	73	33	16	256	9,027	4,556	64	36,500
Dec'99	1,339	381	17	2,544	73	35	8	256	9,167	4,905	64	40,000

NOTE: Averages and standard deviations (SDs) are sales-weighted.

Table 4. Matched-Model Indexes

Period	n(L)	Laspeyre	Paasche	Fisher	n	Chained Laspeyre	Chained Paasche	Chained Fisher
Aug'97	NA	100.0	100.0	100.0	NA	100.0	100.0	100.0
Sep'97	425	96.0	96.2	96.1	425	96.0	96.2	96.1
Oct'97	353	91.2	91.7	91.5	405	91.9	92.6	92.3
Nov'97	294	82.5	81.1	81.8	412	85.5	84.1	84.8
Dec'97	266	77.6	76.6	77.1	400	81.3	79.5	80.4
Jan'98	253	70.3	72.0	71.2	405	75.8	73.8	74.8
Feb'98	206	67.3	67.8	67.5	416	69.7	69.7	69.7
Mar'98	198	60.0	59.8	59.9	431	67.0	66.0	66.5
Apr'98	172	59.0	63.2	61.0	473	62.6	61.9	62.2
May'98	122	56.8	62.5	59.6	427	58.8	58.6	58.7
Jun'98	147	50.7	56.0	53.2	439	53.8	52.6	53.2
Jul'98	88	49.7	53.6	51.6	429	49.3	48.8	49.1
Aug'98	83	50.6	46.8	48.7	443	46.1	46.0	46.1
Sep'98	91	65.1	43.7	53.3	453	43.7	43.3	43.5
Oct'98	103	44.9	44.9	44.9	478	39.7	40.4	40.1
Nov'98	49	57.3	68.9	62.8	429	37.1	37.6	37.4
Dec'98	73	36.4	41.3	38.8	459	35.5	35.8	35.7
Jan'99	45	53.3	45.0	49.0	447	31.8	33.3	32.5
Feb'99	27	77.9	59.7	68.2	419	30.4	32.3	31.3
Mar'99	37	87.9	98.2	92.9	449	29.0	30.6	29.8
Apr'99	12	86.9	105	95.7	471	27.7	29.1	28.4
May'99	4	16.3	33.5	23.4	426	26.6	28.0	27.3
Jun'99	13	14.4	47.7	26.2	459	24.9	26.2	25.6
Jul'99	1	65.5	65.5	65.5	468	23.0	24.6	23.8
Aug'99	2	23.9	47.9	33.8	459	21.8	22.9	22.4
Sep'99	1	65.4	65.4	65.4	473	20.8	21.6	21.2
Oct'99	2	62.3	61.2	61.7	499	19.2	20.6	19.9
Nov'99	2	58.3	58.9	58.6	509	18.1	19.8	18.9
Dec'99	5	13.1	20.1	16.3	532	17.7	19.5	18.5

NOTE: n(L) is the number of units used to construct the Laspeyre index. n is the number of units used to construct the chained indexes.

with the chained indexes there is still a potential selection problem with respect to which products remain in the market from period to period.

Table 5 shows standard hedonic indexes calculated using the same dataset. In implementing the hedonic indexes, we needed to choose a baseline functional form. In the spirit of nonpara-

Table 5. Standard Hedonic Indexes

Period	n(L)	Laspeyre	Paasche	Fisher	n(CL)	Chained Laspeyre	Chained Paasche	Chained Fisher
Aug'97	NA	100.0	100.0	100.0	NA	100.0	100.0	100.0
Sep'97	550	92.9	92.5	92.7	541	92.9	92.5	92.7
Oct'97	550	86.0	86.4	86.2	545	86.5	86.8	86.6
Nov'97	550	84.4	82.5	83.4	502	83.9	83.2	83.5
Dec'97	550	79.3	78.1	78.7	518	78.3	77.9	78.1
Jan'98	550	67.7	69.0	68.3	561	68.1	67.3	67.7
Feb'98	550	70.3	67.6	69.0	517	67.7	67.0	67.3
Mar'98	550	68.4	66.2	67.3	594	65.0	64.4	64.7
Apr'98	550	66.5	63.5	65.0	583	63.4	62.9	63.1
May'98	550	59.3	59.5	59.4	533	57.7	57.4	57.5
Jun'98	550	59.1	52.8	55.9	642	53.4	53.4	53.4
Jul'98	550	56.1	45.2	50.4	547	49.6	50.6	50.1
Aug'98	550	51.3	40.3	45.5	600	45.4	45.9	45.6
Sep'98	550	57.7	39.7	47.9	635	43.5	43.9	43.7
Oct'98	550	47.7	35.4	41.1	635	39.2	40.6	39.9
Nov'98	550	58.3	32.6	43.5	550	38.1	38.5	38.3
Dec'98	550	43.9	28.5	35.4	624	34.6	34.8	34.7
Jan'99	550	49.1	30.5	38.7	579	32.0	32.5	32.3
Feb'99	550	53.7	31.1	40.8	556	31.7	32.4	32.0
Mar'99	550	52.2	30.1	39.6	659	30.9	31.9	31.4
Apr'99	550	47.9	32.1	39.2	622	29.8	30.3	30.0
May'99	550	32.0	31.6	31.8	593	27.2	27.7	27.5
Jun'99	550	24.8	27.6	26.1	663	23.9	24.4	24.1
Jul'99	550	29.3	26.1	27.7	588	22.9	23.3	23.1
Aug'99	550	30.1	21.9	25.7	588	21.3	21.5	21.4
Sep'99	550	27.8	19.5	23.3	646	19.4	19.4	19.4
Oct'99	550	25.4	18.9	21.9	677	18.9	18.7	18.8
Nov'99	550	26.2	17.5	21.4	626	17.9	17.7	17.8
Dec'99	550	16.5	16.4	16.4	705	16.8	16.7	16.7

NOTE: The functional form is semilog. R^2 ranges from .40 to .78. n(L) is number of units used to construct the Laspeyre index. n(CL) is number of units used for the chained Laspeyre index.

metric estimation, when choosing the baseline functional form, our goal was to find the functional form that provided the best fit for the hedonic surface. We tried several functional forms, including linear, semilog [in which only the left side (price) is in log form] and log–log. An analysis of the residuals from these functional forms over several time periods revealed that log-log provided a very poor fit. The linear form fit the high-end machines well, but did not fit the low-end machines well (vastly underpredicting price). The semilog fit the low-end machines well, but showed some slight problems at the high end (slightly underpredicting price). We judged that the best of the three forms was the semilog, and so we proceeded using this as our baseline form. Our judgement was based on a series of statistical tests as well as "eyeing" the fit via residual charts. These results are also consistent with the arguments of Diewert (2003), which argues that the left-side variable in hedonic regressions should be in log form.

Coefficients in the hedonic regressions generally had the expected signs, the main exception being the modem variable, which consistently was estimated to have a negative coefficient. We speculate that this may be due to the fact that computers with modems are generally intended for home use and may be of lower average quality in other respects that are not observed. Individual coefficients are not reported, because there were 29 regressions with approximately 26 coefficients each, for a total of 754 coefficient estimates.

The standard hedonic index and the matched-model index are quite different over some ranges (e.g., from August 1997 to September 1997), probably reflecting the selection problem in the matched-model indexes. However, their movement over the whole sample is surprisingly similar. This result is contrary to the results of Pakes (2003), who found that the selection problem is so bad that the matched-model indexes actually rise for some periods instead of falling. We do not know for sure why our results differ so much in this respect; however, we speculate that selection is not nearly as bad a problem in monthly data as it is in yearly data. In our data, it is typical for more than 90% of the products in one month to be observed in the next month, whereas typically fewer than 10% are observed 12 months later. (However, the fact that 90% of the products are observed from one month to the next does not preclude selection being a bad problem.) From a policy standpoint, this evidence may suggest that it is worthwhile to use higher-frequency data in industries that have a lot of product turnover. Note that our results also show slightly faster rates of decline than those of Aizcorbe et al. (2003) for the period in which the data overlap.

We found that the standard (i.e., nonchained) hedonic indexes were subject to some variability with respect to changing the functional form of the hedonic price function. This variability arises because of the fact that the product space is changing over time. Because PCs are improving over time, when calculating the price index for periods that are far apart in time, it is typically necessary to extrapolate the hedonic price function outside the range of characteristics space on which it was estimated. We found that this introduced substantial variability into the index to the point where we are not confident in the results of the nonchained indexes for even the best of the functional forms. On the other hand, changing the functional form had very little effect on the chained indexes, because very little extrapolation was needed between adjacent periods.

Despite the fact that our dataset contains many characteristics, we found that the R^2 statistics in the hedonic regressions ranged from .40 to .78. Although these were lower than expected, they are in the same range as those given by Pakes (2003), and they are lower than those of Holdway (2001). However, Holdway (2001) used data obtained solely from large web-based retailers and hence likely to be holding many unobserved factors constant. This result suggests that either there are still some important characteristics, such as sales outlet or quality, that we do not observe or there is substantial measurement error in prices.

Table 6 gives standard hedonic indexes similar to those described earlier, except that a polynomial series was used on the right side (as suggested in Pakes 2003), but retaining the semilog form. In general these indexes resulted in better in sample fit of the hedonic function, particularly for those periods in which fit was previously the poorest. Many of the coefficients on the second-order terms were also statistically significant. However, this improvement came at some cost with respect to prediction near the boundaries of the sample in characteristics space. The result is that even with just a second-order polynomial, there are some wild fluctuations in the standard price indexes (see the Paasche price index for July–December 1999). We found that going to higher-order polynomials further improves the fit of the model, but makes the price index even wilder. Again, the problem was not as bad for the chained indexes, as can be seen in the table.

Because of the unreliability of the nonchained indexes here and earlier, in the next section we report results only for the chained indexes. For similar reasons (sampling error), we use the second-order polynomial indexes, rather than the third-order ones, as our base case index.

4.2.2 The Multidimensional Case. Table 7 reports p-values for hypothesis tests based on the GMM objective function for various values of L. The first three columns report the baseline tests run for iid measurement error and three subsamples of approximately 10 periods that divide the data into roughly three pieces. We chose to use 10 period subsamples because 10 periods provided sufficient degrees of freedom to run tests for values of L up to about five while maintaining sufficient observations in the balanced panel. Tests with smaller and larger subsamples generated similar results, as did tests using different subsamples of 10 periods. Results from the three tests are slightly inconsistent, with the early periods requiring a higher dimensional unobservable, but generally imply that the true dimension L is greater than or equal to four.

Because the coverage of the data changes slightly over the panel, and because the variance in PC prices generally falls over our sample, we were concerned that the measurement error variance might not be constant over time. Thus, we reran the hypothesis tests, allowing for heteroscedasticity of unknown form in the measurement error. Relative to the baseline tests, to allow for heteroscedasticity of unknown form, we need only run the same estimation procedure, but throwing out the moment conditions corresponding to the diagonal of the covariance matrix in (16). Allowing for heteroscedasticity of unknown form, all three tests suggest that we cannot reject that the true value is $L = 3$. If we allow L to be large, then it is impossible to test whether the measurement error is heteroscedastic or not,

Table 6. Nonparametric Hedonic Indexes

	Order 2			Order 2				Order 3		
Period	n(L)	Laspeyre	Paasche	n(CL)	Chained Laspeyre	Chained Paasche	Chained Fisher	Chained Laspeyre	Chained Paasche	Chained Fisher
Aug'97	NA	100.0	100.0	NA	100.0	100.0	100.0	100.0	100.0	100.0
Sep'97	550	92.4	92.2	541	92.4	92.2	92.3	92.6	92.7	92.6
Oct'97	550	84.5	85.4	545	85.1	85.5	85.3	85.7	86.4	86.0
Nov'97	550	82.1	80.6	502	81.6	80.4	81.0	81.5	80.9	81.2
Dec'97	550	77.6	75.8	518	76.7	75.4	76.0	76.6	76.2	76.4
Jan'98	550	64.6	66.9	561	67.7	66.2	66.9	67.7	68.2	67.9
Feb'98	550	66.8	64.3	517	66.0	64.3	65.1	65.5	65.8	65.6
Mar'98	550	64.6	62.3	594	62.9	61.2	62.0	62.0	62.7	62.4
Apr'98	550	63.5	59.2	583	60.1	58.5	59.3	59.5	60.0	59.8
May'98	550	56.4	54.6	533	54.9	53.6	54.3	54.4	54.9	54.7
Jun'98	550	56.0	50.1	642	51.6	51.3	51.4	51.2	52.3	51.8
Jul'98	550	54.7	45.5	547	47.4	48.2	47.8	47.1	49.6	48.3
Aug'98	550	50.3	40.0	600	43.5	43.5	43.5	43.2	44.5	43.8
Sep'98	550	59.2	38.8	635	40.9	41.0	41.0	40.6	41.9	41.3
Oct'98	550	47.4	41.5	635	36.9	38.0	37.4	36.5	38.3	37.4
Nov'98	550	60.5	38.9	550	35.4	35.6	35.5	35.3	39.7	37.4
Dec'98	550	46.8	38.5	624	31.9	32.0	32.0	32.7	36.9	34.7
Jan'99	550	53.9	35.7	579	29.7	29.7	29.7	30.6	34.2	32.4
Feb'99	550	63.2	36.1	556	28.9	28.7	28.8	29.7	33.1	31.4
Mar'99	550	59.4	35.9	659	28.0	27.7	27.9	28.8	32.1	30.4
Apr'99	550	54.2	36.0	622	27.6	26.8	27.2	28.4	31.2	29.8
May'99	550	31.2	32.0	593	25.8	25.0	25.4	26.2	28.9	27.5
Jun'99	550	20.6	31.2	663	22.8	22.1	22.5	22.9	25.2	24.0
Jul'99	550	23.8	13,612,488	588	21.9	21.0	21.5	22.1	24.1	23.1
Aug'99	550	23.7	48.9	588	20.6	19.6	20.1	20.8	22.4	21.6
Sep'99	550	22.1	523.9	646	18.7	17.8	18.2	18.8	20.3	19.6
Oct'99	550	24.8	2,153.1	677	18.0	16.9	17.4	18.0	19.4	18.7
Nov'99	550	25.0	2,115.6	626	17.2	16.1	16.6	17.1	18.3	17.7
Dec'99	550	13.4	5,073.8	705	16.2	15.3	15.7	16.2	17.6	16.8

NOTE: The functional form is semilog with polynomial series. R^2 ranges from .50 to .79. n(L) is the number of units used to construct the Laspeyre index. n(CL) is the number of units used for the chained Laspeyre index. The order is polynomial order.

because the model can always match the data equally well by increasing L. However, in our opinion, given the comprehensiveness of the characteristics data, the result that $L = 3$ is more reasonable than those discussed earlier. We also find further support for heteroscedastic measurement error later.

We also worried that the large-sample tests may overreject due to the fact that first-stage estimates are used in place of the true error terms in the hypothesis tests. Because our data did not contain sufficient data points to allow joint estimation of the two stages, we instead simulated finite-sample critical values using the coefficient estimates obtained later and the assumption that both the unobserved characteristics and the measurement error were normally distributed. The results of these tests showed that the finite-sample critical values were indeed larger than the asymptotic critical values. However, as shown in Table 7, the hypothesis that $L = 1$ is still rejected in all cases. We therefore conclude that in this dataset, $L \in \{2, 3\}$.

Table 7. p-Values for Dimensionality Tests

	Homoscedastic ME			Heteroscedastic ME		
Subsample (months):	1–10	11–20	21–29	1–10	11–20	21–29
Large-sample test: Dimension (L)						
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	.046
3	0	.001	.006	.147	.913	.958
4	.001	.244	.749	.757	.963	
5	.025	.248				
Small-sample tests: Dimension (L)						
0	0	0	0	0	0	0
1	0	0	0	0	.003	0
Number of observations	93	58	137	93	58	137

NOTE: The blank areas indicate too few degrees of freedom to calculate.

Table 8 gives chained price indexes constructed for the $L = 0$ and $L = 1$ cases. We estimated the $L = 1$ case using the moments from all balanced panels of length 3, 4, 5, 9, and 10 periods. We found that the results were extremely stable over choices of which panel lengths to use (to within .1 in the overall index). The primary reason for choosing these period lengths was a trade-off between efficiency and computational burden. The more period lengths that we use, the more efficient the estimates, but at the expense of higher computational burden. We wanted to include several short panels because they generally have more data points, as well as several long panels so as to incorporate information from periods that are far apart in the data. Comparing the $L = 0$ and $L = 1$ cases, we find that correcting for unobserved characteristics substantially reduces the index, by 2.9% over the 29-month period for the Fisher index. The reason for this bias is primarily selection. We find that the unobserved characteristics are substantially improving over time (i.e., their normalized mean moves steadily upward from approximately $-.3$ to $.2$ over the period). The standard hedonic index ($L = 0$) absorbs the mean unobserved characteristic in each period into the intercept of the price function. Then when predicting the prices of goods from previous periods that were not observed in later ones, it overpredicts their prices, raising

Table 8. Chained Price Indexes for $L = 0$ and $L = 1$

Dimension	$L = 0$			Homoscedastic M.E. $L = 1$			Heteroscedastic M.E. $L = 1$		
Period	Chained Laspeyre	Chained Paasche	Chained Fisher	Chained Laspeyre	Chained Paasche	Chained Fisher	Chained Laspeyre	Chained Paasche	Chained Fisher
Aug'97	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Sep'97	92.4	92.2	92.3	93.9	93.8	93.8	93.1	93.1	93.8
Oct'97	85.1	85.5	85.3	86.7	87.0	86.9	85.8	86.3	86.9
Nov'97	81.6	80.4	81.0	82.1	81.0	81.5	81.1	80.3	81.4
Dec'97	76.7	75.4	76.0	76.8	75.6	76.2	75.9	74.9	76.1
Jan'98	67.7	66.2	66.9	70.5	69.6	70.0	70.2	69.5	70.0
Feb'98	66.0	64.3	65.1	66.2	65.2	65.7	65.2	64.4	65.6
Mar'98	62.9	61.2	62.0	61.7	61.0	61.3	60.9	60.4	61.2
Apr'98	60.1	58.5	59.3	58.4	57.8	58.1	57.6	57.2	58.0
May'98	54.9	53.6	54.3	53.8	53.4	53.6	53.2	53.0	53.7
Jun'98	51.6	51.3	51.4	49.4	50.2	49.8	48.9	49.9	49.8
Jul'98	47.4	48.2	47.8	45.0	46.9	46.0	44.5	46.5	45.9
Aug'98	43.5	43.5	43.5	41.3	42.4	41.8	40.9	42.2	41.8
Sep'98	40.9	41.0	41.0	37.6	38.6	38.1	37.0	38.2	38.0
Oct'98	36.9	38.0	37.4	33.5	35.4	34.5	33.5	35.5	34.4
Nov'98	35.4	35.6	35.5	30.9	31.7	31.3	30.5	31.4	31.3
Dec'98	31.9	32.0	32.0	28.2	29.2	28.7	28.2	29.1	28.7
Jan'99	29.7	29.7	29.7	24.9	25.9	25.4	25.1	25.9	25.6
Feb'99	28.9	28.7	28.8	23.2	24.1	23.7	23.2	24.1	23.9
Mar'99	28.0	27.7	27.9	22.4	23.5	22.9	22.4	23.6	23.1
Apr'99	27.6	26.8	27.2	21.8	22.5	22.2	21.8	22.4	22.4
May'99	25.8	25.0	25.4	20.1	20.8	20.5	20.0	20.6	20.6
Jun'99	22.8	22.1	22.5	18.1	18.6	18.4	18.1	18.6	18.6
Jul'99	21.9	21.0	21.5	17.1	17.5	17.3	17.0	17.4	17.5
Aug'99	20.6	19.6	20.1	16.0	16.3	16.2	16.0	16.2	16.3
Sep'99	18.7	17.8	18.2	14.6	14.8	14.7	14.6	14.7	14.8
Oct'99	18.0	16.9	17.4	13.8	14.0	13.9	13.8	13.9	14.0
Nov'99	17.2	16.1	16.6	13.0	13.1	13.1	13.0	13.1	13.2
Dec'99	16.2	15.3	15.7	12.8	12.9	12.8	12.8	12.9	12.9

the overall index. Most of this movement in the average unobserved characteristic takes place during the first 18 months in the data, which is also reflected in the estimated indexes.

We found that allowing for heteroscedastic measurement error did significantly affect the parameter estimates, and, based on this, we feel that this is the correct specification. However, we found that it had very little impact on the overall price index. The primary effect was to increase the rate of price deflation in early periods, and slow the rate of price deflation midway through the sample.

Unfortunately, we found that we were unable to estimate the price index for $L > 1$ cases in our data, because it was not possible to estimate a two-dimensional ξ very precisely for many products. We found that there was so much noise in the estimates of ξ that the price index calculations were also too noisy to be reliable. Thus, because we found that $L \in \{2, 3\}$ earlier, the $L = 1$ case must be viewed as an approximation to the true index. In view of the comprehensive nature of our data, this finding leads us to be slightly pessimistic with respect to the ability to correct for multiple unobserved factors in other industries.

However, there are several factors that also make these data more difficult to work with than data from other industries. One factor is the high rate of product turnover, which leads to products often being observed only in a handful of periods. If products are observed more often, then estimation of multiple factors becomes easier. Another factor is the fact that our data were not collected as carefully as that of the BLS. With less measurement error, estimation would also be easier.

4.2.3 Omitting an Important Characteristic. Table 9 tests our approach for the case when an important characteristic, the CPU benchmark, is known to be omitted. We compare the estimated price index constructed using all of the observed product characteristics (as before) against those constructed using all of the observed characteristics except CPU benchmark. This experiment tests our approach's ability to reduce the bias from unobserved product characteristics for a case in which the bias is quantifiable. Note that in our model, leaving out CPU benchmark is equivalent to leaving out several characteristics, because our base case model uses a second-order polynomial in all of the continuous characteristics.

The first two columns of Table 9 provide standard chained Fisher indexes first including all characteristics and then omitting CPU benchmark. As indicated earlier, the results show that a substantial bias occurs in this case. Over the entire period, the difference between the two indexes is approximately 9%.

The third and fourth columns report the same results after controlling for the unobserved characteristics. The two indexes do not agree entirely, but are much closer than the two standard indexes. Over the entire sample, the difference is now just 2.3%. In fact, the first, third, and fourth columns of the table are remarkably similar.

We also calculated the correlation between the estimated values of ξ and the left-out CPU benchmark variable to see whether the estimated ξ's reflected the left-out characteristic. We found that the correlation with CPU benchmark was .41. We take this as evidence that the procedure is working the way it is supposed to. Note that when CPU benchmark is omitted, the unobserved characteristics pick up only the residual correlation of CPU benchmark with prices once the effects of RAM

Table 9. Chained Price Indexes: Multidimensional Cases With and Without the CPU Benchmark

	Chained Fisher indexes			
	Standard indexes		Corrected indexes	
Period	All characteristics	CPU omitted	All characteristics	CPU omitted
Aug'97	100.0	100.0	100.0	100.0
Sep'97	92.3	93.4	93.8	93.6
Oct'97	85.3	86.8	86.9	86.8
Nov'97	81.0	83.9	81.4	82.8
Dec'97	76.0	78.8	76.1	77.2
Jan'98	66.9	68.9	70.0	69.6
Feb'98	65.1	68.4	65.6	67.8
Mar'98	62.0	65.8	61.2	64.3
Apr'98	59.3	64.3	58.0	61.4
May'98	54.3	59.1	53.7	57.2
Jun'98	51.4	56.4	49.8	53.0
Jul'98	47.8	53.0	45.9	48.9
Aug'98	43.5	49.5	41.8	45.7
Sep'98	41.0	47.7	38.0	41.8
Oct'98	37.4	43.7	34.4	38.3
Nov'98	35.5	42.3	31.3	34.8
Dec'98	32.0	38.5	28.7	32.2
Jan'99	29.7	37.0	25.6	29.2
Feb'99	28.8	37.1	23.9	27.9
Mar'99	27.9	37.5	23.1	27.7
Apr'99	27.2	37.3	22.4	26.4
May'99	25.4	34.9	20.6	23.8
Jun'99	22.5	30.4	18.6	21.3
Jul'99	21.5	30.7	17.5	20.3
Aug'99	20.1	29.5	16.3	19.1
Sep'99	18.2	27.6	14.8	17.3
Oct'99	17.4	27.4	14.0	16.5
Nov'99	16.6	26.5	13.2	15.6
Dec'99	15.7	25.1	12.9	15.2
Average n	573	573	573	573

NOTE: Corrected indexes are robust to heteroscedastic measurement error.

and hard drive and the other characteristics are accounted for. Thus we view the .41 figure as being quite high.

In theory, with sufficient data, the procedure should provide the same results whether or not CPU is included. So what explains the fact that the results are not 100% consistent? Part of the difference between the two is almost surely explainable by the fact that we were unable to estimate the index for higher values of L. The procedure involves using a one-dimensional unobservable to try to match the previous results ($L \in \{2, 3\}$), plus now CPU benchmark is left out, so we should expect that $L \geq 4$. Thus the procedure relies on an approximation. A second reason for the difference would be if the relationship between CPU benchmark and the other observed characteristics among the observed products is changing substantially over the sample period.

5. CONCLUSIONS

In this article we have presented both theoretical and empirical evidence demonstrating that omitted product characteristics can lead to a severe bias in hedonic price indexes. Moreover, we have shown that, at least for our data on desktop PCs, this bias is of practical relevance. In the case of PCs, we have found evidence of a selection bias in the standard hedonic index that biases the index upward by about 1.4% per year in our sample.

Given the comprehensiveness of the characteristics data available for PCs, we found it somewhat surprising that the unobserved characteristics bias was this large. In other industries where hedonic techniques are currently used by the BLS, such as housing and apparel, we might expect collecting such comprehensive data to be more difficult, leading to important unobserved characteristics. On the other hand, mitigating this effect is the fact that unobserved characteristics in these industries are likely to change less quickly over time, reducing the selection bias.

We have also presented an approach for constructing hedonic indexes that controls for unobserved product characteristics under quite general assumptions. This approach can be viewed as a middle ground between the standard hedonic approach and the matched-model approach. The drawback of our approach is that it requires more data than the standard hedonic approach, because it requires data on the same products over several time periods. However, its data requirements fall far short of those of the matched-model approach. Our methodology also shows how to do factor analysis more generally for unbalanced panels when selection is present.

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