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A DYNAMIC MODEL OF HOUSING DEMAND: ESTIMATION AND POLICY IMPLICATIONS*

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Using data from the PSID, we estimate a dynamic model of housing demand with nonconvex adjustment costs, credit constraints, and uncertainty about income and home prices. We simulate how consumer behavior responds to house price and income declines as well as tightening credit. In response to a negative home price shock, households early in the life cycle climb the housing ladder more quickly and invest more in housing assets due to the lower price. With a concurrent negative income shock, however, housing demand falls among young and middle aged households who stay in smaller homes rather than to trade up.

1. INTRODUCTION

In this article, we estimate and simulate a dynamic structural model of consumer demand for housing and nondurable consumption. We use our dynamic model to simulate a typical household's response to a set of negative shocks meant to mimic the current disruptions in the U.S. economy. The counterfactual experiments we consider are a decline in housing prices, a decrease in income, and a tightening of lending standards. Our results have important implications for understanding the impact on housing demand of the recent house price and income shocks that hit the U.S. economy.

In the decade prior to the housing bust, home prices had appreciated at a very fast rate compared to historical standards. Between 1997 and 2006, the nationwide Case Shiller home price index more than doubled from 84 to 190. The rate of appreciation in certain U.S. cities was much faster than the national average. However, home prices sharply dropped, and there is little evidence thus far that there will be a quick recovery. Between the peak in 2006 Q2 and 2009 Q2, the nationwide index fell 30%. For the most hard hit cities, such as Miami, Detroit, San Diego, Las Vegas, and Phoenix, prices have fallen between 45% and 55% as of June 2009. Accompanying the home price drop, the real economy entered the most severe recession since WWII, resulting in increased unemployment and a decline in household incomes. In our counterfactual exercises, we expose households to sudden drops in both home prices and income of the form observed in the data.

The past two decades have also seen substantial changes in mortgage markets. The traditional 30 year fixed rate mortgage is no longer the standard mortgage product. Since their introduction

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in the early 1980s the adjustable rate mortgage had grown to a recent peak of a 40% share of mortgage applications in 2005.[2] There was also an expansion of the subprime mortgage market and other nonconforming loans. These credit market innovations helped people with low-credit quality become homeowners and also allowed households to buy larger homes. However, it is now clear that the expansion of subprime credit also had a downside. Approximately 15% of subprime loans were in default as of August 2009, three times the rate in 2005.[3] As a result, nonconforming mortgages have been more difficult to obtain as lenders have tightened credit in the mortgage market.[4] Our structural model is rich enough to trace out, in an admittedly stylized way, the consequences of a contraction in the ability of households to collateralize debt with housing.

In our structural model, a household solves a life-cycle dynamic programming problem. In each period, households make investment decisions in housing, choose nonhousing consumption levels, and make decisions regarding mortgage borrowing and savings. Unlike a typical investment vehicle, housing provides a flow of services that enters utility along with nonhousing consumption. We include additional realistic features of housing demand in the model. Adjusting the stock of housing requires the household to incur transaction costs meant to capture realtor fees and other costs related to buying and selling a home. This gives rise to a lumpy pattern in housing investment. The model also includes credit constraints in the form of minimum down-payment requirements for mortgages. We also allow households to carry zero-housing assets, and instead participate in a frictionless rental market. Finally, in order to assure that we can account for the observed heterogeneity in the housing tenure decisions in the data among households with similar observable characteristics, we include preference shocks to the utility from owner occupied housing services into the model. By modeling the evolution of incomes and home prices as exogenous first order Markov processes we adopt a partial equilibrium modeling strategy.

We estimate our model using household level data on income, wealth, and housing tenure decisions from the Panel Study of Income Dynamics (PSID). We solve for the structural parameters in the the model using the multistep method proposed by Bajari et al. (2007; hereafter BBL). The first step of BBL requires us to estimate housing decision rules, savings decision rules, and the law of motion for the state variables. We use an ordered probit model of housing investment decisions in the spirit of Han (2010). In each period households either upgrade to a larger home, remain in their existing home, downgrade to a smaller home, or rent with zero-housing investment. This reduced form model provides a flexible way to capture lumpy patterns in housing investment in the presense down-payment constraints. We estimate the evolution of the exogenous state variables using standard time series and panel econometric techniques. In the second step, we estimate the structural parameters in household utility. The estimator proposed by BBL solves a revealed preference problem. We assume that the policy functions estimated for housing investment and savings in the first stage are the solution to a household's dynamic programming problem. The estimator reverse engineers a period utility function that rationalizes the estimated decision rules by solving a system of revealed preference inequalities. An attractive feature of our two-step method is that it allows us to estimate a model with nonconvex adjustment costs and credit constraints without ever having to directly solve the dynamic programming problem. Computing a single optimal-policy function in our model takes one week of CPU time with an advanced workstation. Our estimator avoids the burden of repeatedly computing optimal policies.

Given our parameter estimates, we use our dynamic model to simulate a typical household's response to a set of negative shocks to home prices, income, and down-payment requirements meant to mimic the current disruptions in the U.S. economy. Our results show a muted response in housing investment in the short run because of the adjustments costs. Absent adjustment costs, households make immediate and continuous adjustments. But over the long run there are

[2] Source: Freddie Mac annual ARM (adjustable rate mortgage) survey.

[3] Source: Mortgage Bankers Association.

[4] According to the 2009 Freddie Mac annual ARM survey, the share of ARM applications is at an all time low of 3%.

substantial effects. In response to a home price decline, young households that are climbing the housing ladder upgrade their homes earlier in life and invest in more housing because of the lower price. In this sense the substitution effect dominates the wealth effect of decreased existing home equity. But with a concurrent income shock those households remain in smaller homes and thus we see a substantial negative effect on housing demand. Down-payment constraints are important for understanding housing demand over the life cycle. Even with a strong preference for owner occupied housing, young households choose to rent first instead of purchasing a suboptimally small home. While being renters they save for a down payment on a home of larger size. Tighter down-payment constraints extend the time households remain renters. We also perform a sensitivity analysis to show that the utility function parameters we estimate have significant quantitative implications for our simulations. In particular, we estimate an elasticity of substitution between housing and nondurable consumption that is higher than the unit elasticity often used in the literature. In our simulations we demonstrate that the estimate of the size of this parameter matters in a quantitatively significant way, with a lower value mainly delaying home ownership over the life cycle.

The article is organized as follows. We review the theoretical and empirical literature relevant for our study in the next section. Section 3 describes the model. Section 4 presents the data and descriptive evidence. In Section 5, we provide an overview of the BBL estimation technique. Section 6 and Section 7 describe the first and second stages of estimation. In Section 8, we present our benchmark model simulation results, and Section 9 conducts sensitivity analysis with respect to crucial structural parameters. Section 10 concludes; additional details about computation are relegated to the appendix.

2. RELATED LITERATURE

Following Mankiw's (1982) aggregate study of consumer durables, a sizable literature has developed that uses structural household-level models of housing demand and tenure choice to study the interaction between house prices, household consumption, and tenure decisions.

The closest papers in spirit to ours are the works by Li and Yao (2007) and Li et al. (2009). The former paper constructs a life-cycle model of housing tenure choice to study the impact of house price changes on housing choices and consumer welfare for different age groups of the population. The key distinction between their paper and ours is twofold. First, although their paper contains a more explicit model of the life-cycle and housing tenure choice, they restrict attention to a preference specification in which the elasticity of substitution between nondurables and housing services is fixed to one (that is, the aggregator is of Cobb–Douglas form). Our article, as in Li et al. (2009) *estimates* this crucial parameter and finds the elasticity to be larger than one. Our model simulations document that this difference has important implications for the dynamics of housing and consumption choices. Second, although their main focus lies on the impact of house price changes on consumption allocations over the life cycle and the distribution of its welfare impacts across different households, we focus more directly on the impact of house price shocks on housing demand and the demand for nondurable consumption.

We estimate the structural parameters of our dynamic model using the two-step method of BBL. Alternative methods include method of moments estimators as used in Li et al. (2009) and Euler equation approaches as in Hansen and Singleton (1982). Full solution methods include the nested fixed point algorithm in Rust (1987) and the MPEC constrained optimization approach in Judd and Su (2012). Our paper also contributes to the literature on the estimation of dynamic decision problems with nonconvex adjustment costs. To the best of our knowledge, the only paper that attempts to estimate such a structural model, besides Li et al. (2009) cited above, is Hall and Rust (2003), which uses a simulated minimum distance estimator. Finally, our article is closely related to Han (2010). Our first stage is quite similar to her reduced form model of housing demand. We depart from her work because we estimate structural utility parameters.

Related to our economic model of housing, Flavin and Yamashita (2002), Fernandez-Villaverde and Krueger (2011), Yao and Zhang (2005), Hintermaier and Koeniger (2011), Kiyotaki et al. (2008), Iacoviello and Pavan (2011), and Diaz and Luengo-Prado (2008, 2010) use a similar life-cycle model to study the impact of the presence of housing on portfolio choice, precautionary saving, and the wealth distribution. The latter authors also employ their model to argue that the current user cost approach to measure price changes for housing services in the consumer price index (CPI) is biased in the presence of owner occupied housing and household heterogeneity. In a sequence of quantitative papers Chambers et al. (2009a, 2009b, 2009c) use a life-cycle model with tenure choice to explore the impact of tax treatments and different mortgage designs on home ownership rates and explore the reasons for the substantial increase in this rate in the United States in the last decade. Yang (2009) documents the role of down-payment constraints and transaction costs on the life-cycle profile of housing and consumption. Ortalo-Magne and Rady (2006) study the interaction of financial markets conditions and home ownership rates in a structural model of housing choices. Lustig and van Nieuwerburgh (2006) and Piazessi et al. (2007) explore the connection between house and asset prices.

The role of housing as collateral is the main theme of a recent literature that focuses on the joint housing and mortgage choice. Important examples of this work include Hurst and Stafford (2004), Luengo-Prado (2006), and Chambers et al. (2009a, 2009b). The recent increase in default on mortgages has motivated a small but growing literature on structural models of foreclosures within this context. See, for instance, Jeske et al. (2011) and Garriga and Schlagenhauf (2009). The same issue is analyzed empirically, among others, by Carroll and Li (2011).

On the empirical side, an attempt has been made to quantify the wealth effect on nondurable consumption from changes in house prices. Leading work in this strand of the literature include Case et al. (2005), Benjamin et al. (2004), and Campbell and Cocco (2007). They document a sizable housing wealth effect and contrast their results to estimates of the wealth effects from other financial assets. We use our model to document how strongly nondurable consumption responds to a house price decline and study how this negative wealth effect from house prices interacts with down-payment constraints and negative income shocks.

3. THE MODEL

We model a typical household's consumption and housing choice as a partial equilibrium, dynamic decision problem with a finite lifetime horizon. Households live for T periods, and in each period t they choose consumption expenditures on nondurables, c_t, and the amount of one-period risk-free financial assets to bring to the next period, a_{t+1}. We let h_t denote the size of the household's owned real housing stock brought into the period, so that h_{t+1} is the amount of owned housing chosen for today. Households have the mutually exclusive choices of owning, $o_t = 1$, and renting, $o_t = 0$. Both options deliver housing services, s_t. Let l_t denote the real amount of rented housing. Since households cannot simultaneously be renters and homeowners, we have $h_{t+1} = 0$ for a renter and $l_t = 0$ for an owner. Thus

$$s_t = o_t e^{\kappa_t} h_{t+1} + (1 - o_t) l_t, \tag{1}$$

where κ_t is an i.i.d. preference shock that affects the services derived from owner occupied housing. A positive κ_t realization (and thus $e^{\kappa_t} > 1$) therefore makes owning more attractive relative to renting, all else equal.

Households value nondurable consumption, c_t, and housing services, s_t, according to the period utility function $u(c_t, g^* s_t)$, where $g > 0$ is a parameter. Expected lifetime utility is then given by

$$U\left(\{c_t, h_{t+1}, l_t\}_{t=0}^{T}\right) = E_0\left[\sum_{t=1}^{T} \beta^{t-1} u(c_t, g^* s_t) + \gamma \beta^T \log(b_T)\right], \tag{2}$$

where β is the standard time discount factor, T determines the end of working life, and γ measures the degree of altruism to leave bequests b_T. Apart from bequest motives, the parameter can be interpreted as a preference to carry a positive balance of retirement savings b_T to insure against an unknown life expectancy. Expectations E_0 are taken with respect to the stochastic processes driving labor income, owner occupied housing preference shocks, and house prices, which we specify later.

Let p_t denote the relative price of one unit of housing in terms of the numeraire nondurable consumption good. Housing prices $\{p_t\}_{t=0}^{T}$ follow first order stochastic Markov processes. We assume rental prices p_{lt} are proportional to house prices, that is $p_{lt} = \rho p_t$, where ρ is a fixed parameter.

At time 0, agents are endowed with initial asset holdings (a_0, h_0) and one unit of time per period, which they supply inelastically to the labor market to earn labor income y_t. The labor income process is composed of two components, a deterministic mean life-cycle profile ε_t (which incorporates income growth over the life cycle) and a stochastic component η_t that follows a first order Markov process. Thus labor income is given by $y_t = \varepsilon_t \eta_t$.

We model two main frictions in the housing market explicitly. First, the stock of housing is subject to nonconvex adjustment costs. Specifically, in order to purchase a home of size h_{t+1} the household has to spend

$$p_t h_{t+1} + p_t \phi(h_{t+1}, h_t),$$

where $p_t h_{t+1}$ is the purchase price of the home and $p_t \phi(h_{t+1}, h_t)$ is the transaction cost a household has to bear when adjusting the owned stock housing from h_t to h_{t+1}. We assume that the function ϕ takes the form

$$\phi(h_{t+1}, h_t) = \begin{cases} \phi * h_{t+1} & \text{if } h_{t+1} \neq h_t \\ 0 & \text{if } h_{t+1} = h_t \end{cases},$$

where the number ϕ measures the size of the transaction cost. In most of our analysis we shall assume that $\phi = 0.06$, which is representative of real-estate fees paid by a real estate *buyer*.[5] Note that under this specification the seller bears no transaction costs. This also implies that there are no adjustment costs from moving into renting, $\phi(h_{t+1} = 0, h_t > 0) = 0$, positive adjustment costs to move from renting to owning, $\phi(h_{t+1} > 0, h_t = 0) = \phi h_{t+1}$, and no adjustment costs to move across rentals, $\phi(h_{t+1} = 0, h_t = 0) = 0$. For the purpose of modeling, it would be straightforward to include adjustment costs in rental-to-rental moves, but they are omitted for the purposes of estimation. In the data, we observe very frequent moves among rental units (four times more frequent than moves involving owner occupied housing), which suggests that setting zero-adjustment costs in rental moves is a reasonable approximation.

A second key friction in our model is the requirement for households to acquire (and maintain) some minimal positive equity share in the house. We assume that the joint choice of financial assets and housing positions satisfies the following collateral constraint:

$$a_{t+1} \geq -(1-\xi) p_t h_{t+1}. \tag{3}$$

Here $\xi \in [0, 1]$ is the fraction of the purchase price of the house that has to be paid down at purchase, that is, $(1-\xi)$ is the fraction of the purchase price that can be financed via a mortgage. In most of our experiments we shall assume that households are able to finance at most 80%

[5] We do not have direct data on transaction prices. In principle, we could infer them indirectly through the use of our structural model. However, taking a direct stance on transaction costs will give a more efficient estimate of the remaining model parameters. As we shall show in our policy simulations, our model is able to reasonably match key moments of our data, where transaction costs play a large role. Also, the simulation results suggest that qualitatively, many of our policy conclusions will be robust to a fairly broad range of transaction costs as long as they are sizable (and as long as the adjustment cost function has the nonconvex form we have specified).

of their housing purchases through mortgages.[6] Also note that as long as $\xi \in [0, 1]$, households can only borrow against their housing collateral; uncollateralized debt is therefore ruled out by assumption in our model.

Thus the key frictions in the housing market are summarized by the transaction cost ϕ parameter and the collateral constraint parameter ξ, with $\phi = \xi = 0$ denoting frictionless housing markets. Our simulation exercises will therefore be able to quantify the importance of frictions in the housing market by deducing optimal choices of households under various assumptions on (ϕ, ξ).

In addition to housing, households can use financial assets to accumulate wealth. These assets yield a real interest rate r_t. If households borrow (subject to the collateral constraints), they face a real mortgage interest rate $r^m > r$. In our counterfactual exercises we treat interest rates as constant, but treat them as stochastic for the purposes of estimation.

Defining

$$r(a) = \begin{cases} r & \text{if } a \geq 0 \\ r^m & \text{if } a < 0 \end{cases},$$

the budget constraint can be written as

$$c_t + a_{t+1} + o_t p_t h_{t+1} + (1 - o_t) p_{lt} l_t + p_t \phi(h_{t+1}, h_t) = y_t + (1 + r(a_t)) a_t + p_t h_t. \tag{4}$$

Finally, consumption and housing choices are constrained to be nonnegative:

$$c_t, h_{t+1}, l_t \geq 0. \tag{5}$$

Households maximize (2) subject to the constraints (3), (4), and (5). In the appendix we offer further details on the recursive formulation and computation of this partial equilibrium household decision problem.

4. DATA AND DESCRIPTIVE EVIDENCE

In this section, we discuss our data and present descriptive evidence that motivates our modeling choices.

The household level data come from the Panel Survey of Income Dynamics (PSID), a national household level longitudinal survey. The sample spans the years 1980–1993. We collected time series data on 30-year mortgage interest rates from the Federal Housing Finance Agency[7] and times series data on home prices from the same agency based on home sales financed with Fannie Mae and Freddie Mac conventional mortgages. Both series span the years 1975–2009. We use a longer time series than the PSID data to capture the recent run up and subsequent rapid drop in home prices. All values are in real terms, and dollar values are deflated by the nonhousing component of the consumer price index (CPI) to a base year of 1980.

From the PSID, we select a panel of households aged between 25 and 70. The sample contains 7,316 households for a total of 43,533 household-year observations. The key variables include demographics, income, housing tenure (rent/own), value of owner occupied housing or annual rent payment, annual mortgage payment, home equity, and an indicator for whether the household moved in a given year. We also include nonhousing wealth data from the PSID, but the data are available only for the sample years 1984 and 1989. For the other years, we proxy wealth using the wealth observation from the nearest available year. We drop outliers in terms

[6] This is a typical down-payment requirement in conforming mortgages offered by Freddie Mac and Fannie Mae. Our sample, which dates between 1980 and 1993, predates the explosion in subprime mortgages. A 20% down-payment requirement is representative of credit constraints during this time period.

[7] Formerly the Office of Federal Housing Enterprise Oversight (OFHEO).

of wealth, income, house value, home equity, and rental value. For some of our estimates we need to drop additional observations with missing data in the lagged year.[8]

4.1. *Aggregate Descriptive Evidence.* Table A.1 lists descriptive summary statistics. The table shows statistics for all households and also breaks them down by renters and homeowners for comparison purposes. Housing is a significant component of household expenditures and wealth. The average household occupies a home worth $46,000, approximately double the average income of $23,000. About a third of the households rent in any given year. The average renter is younger, has a smaller family, and has lower income than the average homeowner. Homeowners occupy more valuable homes than renters, in part due to their higher income—homeowners average about double the income of renters—but renters allocate a greater share of income to housing. Renters spend an average of 20.7% of income on rent, whereas the owner's equivalent rent for homeowners places that figure at 16.6%.[9] On average, homeowners have a two-thirds equity stake in their home, and most, 77%, finance using a mortgage. They typically hold other nonhousing financial assets in addition to housing. The split between home equity wealth and nonhousing wealth is about equal. Renters have much less wealth than homeowners in both absolute value and as a share of income; renters have wealth equal to 40% of income, and homeowners, almost triple income. It is also worth noting that some households have slightly negative wealth. The difference in wealth between homeowners and renters matches the down-payment constraint feature of our model: Renters cannot afford a down payment to move into home ownership. To further illustrate, the average renter has barely enough wealth to meet a conventional 20% down-payment requirement to purchase the home it currently rents, just 27%. Using the difference in 1984 and 1989 nonhousing wealth, we impute an annualized measure of nonhousing saving. On average saving is just above zero for both renters and homeowners, slightly higher for renters. But it is worth noting there is significant heterogeneity across households. In particular, we estimate that low-wealth households save more than high wealth households. This pattern is consistent with the life-cycle model that predicts low-wealth households save more aggressively to finance home purchases and to build wealth for bequests. Besides nonhousing wealth, homeowners accumulate wealth by paying down mortgages.

Adjustment costs to moving play a key role in our model. If the costs are high enough, the model predicts households should move in, out, and among owner occupied housing infrequently and make lumpy adjustments to housing stocks. This behavior is evident in the data. Households adjust their stock of owner occupied housing in just 6.5% of the years. Extrapolated over a life cycle between the age of 25 and 70, the average household adjusts about three times. Further breaking down this figure, renters move to home ownership at a relatively high frequency, 6.8%. For homeowners, most moves involve upgrades to larger homes, 2.9%, and they downgrade less frequently, 1.4%. There is a moderate amount of churn in homeowners switching to rental housing, 2.4%. To economize on the number of state variables and parameters in the model, we assume adjustment costs are zero for rental-to-rental moves. As evidence for zero, or near zero-adjustment costs, we see a very high-moving frequency among rentals: 26.8% of renters who remain renters move to a different unit. The size of the adjustments is large. The average upgrading household increases its housing stock by $26,000, and the average downgrading household decreases its stock by $18,000. The large adjustments are consistent with lumpy investment behavior. The large standard deviation in the ratio of house value to income, 2.2, also suggests that households do not continuously adjust housing in response to income fluctuations.

[8] We drop households with income less than $2000 and greater than $120,000, nonhousing wealth less than $10,000 and greater than $400,000, home equity ratios (home equity/house value) less than −0.2, annual rent less than $1,500, and observations that indicate the household is simultaneously a renter and owner. It is also worth noting rent value was not surveyed in 1989, one of the years with the wealth supplement.

[9] We use an owner's equivalent rent ratio of 0.075 for the purposes of describing the data. None of our estimates rely on a specific value of the ratio.

4.2. *Life-Cycle Patterns.* Table A.2 reports summary statistics by birth cohort, which we will interpret as being suggestive of life-cycle behavior.[10] Income, housing, and wealth exhibit a hump-shaped pattern. Young households progressively invest in more housing until they are middle-aged and downsize at older ages. Part of this accumulation can be attributed to households moving out of rental housing and into owned housing. At young ages a high fraction of households rent, approximately 50%, and then the fraction declines rapidly as households age to a trough at 20%. There is a slight increase in renting at older ages as households downgrade their homes. They accumulate wealth in both home equity and nonhousing financial assets through middle age, and then it declines during older ages. But, they do not fully drain wealth by the age of 70. Although off its peak, wealth remains high at the age of 70 relative to wealth at ages below 40. For this reason, we need the bequest motive in our model. The wealth trends can in part explain renting decisions. At young ages, the amount of wealth relative to the value of the home being occupied (whether rented or owned) is low, which suggests down-payment constraints lock households into renting. At the age of 60, wealth and the ratio of wealth to housing peak. The average household has 1.8 times as much wealth as its house value. At this level, down-payment constraints are unlikely to bind, and we see that home ownership rates are the highest at the age of 60.

The bottom of the table shows the frequency of moves at various stages of the life cycle and breaks them down for renters and homeowners. As the aggregate summary statistics show, households move infrequently, but the rates differ by age. For all types of moves, the frequency declines in age, from 11% for the youngest cohort to 3% for the oldest. Most of this trend can be explained by households moving up the housing ladder: both renters moving to home ownership and homeowners upgrading to larger homes. The rate of upgrading is high at young ages and then tapers off as households age. Similarly, homeowners are less likely to downgrade or move to rentals as they age. The downgrading pattern is not quite as pronounced as upgrades, but nonetheless there is an identifiable trend. Perhaps there is an uptick in downgrade frequencies at older ages. It is also interesting to note that there is an appreciable amount of switching from home ownership to renting at all ages. Clearly, home ownership is not an absorbing state. Shocks to such things as preferences, income, and wealth are inducing homeowners to switch to renting. Finally, the frequency of downgrades and switches to renting is relatively high at young ages. This is not consistent with the average trend showing young households rapidly upgrading. Down-payment constraints could explain this pattern. Because many young households have wealth levels near the down-payment constraint, even modest negative shocks to income or home prices could force them to downgrade or switch to renting.

At the very bottom of the table, we report the home equity ratio for the subsample of homeowners. The ratio steadily increases, with no hump-shaped pattern. At young ages households have about a 35% equity stake and by the age of 65 or 70, households have almost no mortgage debt. We will assume households finance with 30-year fixed rate mortgages. This pattern in the data is consistent with the equity accumulation for such mortgages. The data certainly do not indicate households were using mortgages with shorter than 30-year terms, interest-only payment schemes or taking out large home equity lines of credit.

5. ESTIMATION PROCEDURE

We use the two-step method proposed by Bajari et al. (2007) to estimate the structural parameters of our model. The estimation procedure in this article proceeds in two steps. In the first step, the economist flexibly estimates the reduced form decision rules and law of motion for the state variables. In the model in Section 3, the exogenous state variables include income, home prices, and interest rates. Endogenous state variables include the current stock of housing,

[10] In general, a cohort table cannot distinguish life-cycle effects from cohort effects; life-cycle interpretation is only suggestive.

wealth, and housing tenure status. The decision rule is the optimal choice of housing and wealth as a function of the current state.

In the second step, the economist finds the structural payoff utility parameters that rationalize the reduced form decision rules. We parameterize period utility with a familiar constant elasticity of substitution form. During the life cycle, households derive utility from either owner occupied housing, h', or rented housing, l, and nondurable consumption, c, according to the period utility function:

If owner

$$u(c, h' > 0, l = 0) = \log\left[(\theta c^{\tau} + (1-\theta)(\kappa h')^{\tau})^{\frac{1}{\tau}}\right].$$

If renter

$$u(c, h' = 0, l > 0) = \log\left[(\theta c^{\tau} + (1-\theta)(l)^{\tau})^{\frac{1}{\tau}}\right].$$

The parameter θ captures consumption shares, and the parameter τ captures the elasticity of substitution between housing and nonhousing consumption. The elasticity parameter, often treated as unit elasticity ($\tau = 0$), is particularly important, as our counterfactual exercises will demonstrate in the next section. The utility flow of housing parameter, $\kappa > 0$, is stochastic: distributed log-normal i.i.d. across years and households with mean μ_{κ} and variance σ_{κ}^2.

In our model, households maximize lifetime expected discounted utility given time t information. In order to simplify our problem, we assume that households live until the age of 70. Expected discounted utility can be written as

$$E_t \sum_{t'=t}^{70} \beta^{t'-t} u(c_{t'}, h_{t'}) + \beta^{70-t} \gamma \log(a_{70}).$$

In the above, a_{70} is the amount of assets held at the age of 70 to be left as a bequest. The parameter γ describes a household's preference to leave a bequest. A bequest motive is necessary to understand observed choices because otherwise agents would have a strong incentive to liquidate all assets at the age of 70. Besides bequest motives, an alternative interpretation of the parameter is that it reflects precautionary incentives to save for retirement. We do not distinguish between the two motives.

In total, there are five structural parameters to estimate: $\{\theta, \tau, \mu_{\kappa}, \sigma_{\kappa}, \gamma\}$.

6. FIRST STAGE: REDUCED FORM POLICY FUNCTIONS

The first step of Bajari et al. (2007) requires us to estimate an agent's reduced form decision rule. In the model in Section 3, the household chooses how much housing and financial assets to carry into the next period, h_{t+1}, a_{t+1}, nonhousing consumption c_t and, if a renter, the amount of rental housing for the current period l_t. Our dynamic programming model implies that these decisions should be a function of an agent's state variables. We begin by describing our estimation of h_{t+1}.

In the presence of transactions cost to moving, households move infrequently and, when they do, make lumpy adjustments to their stock of housing. We model this as a mixed discrete/continuous choice problem. The method of Bajari et al. (2007) requires that we flexibly approximate the agent's decision rules so that we let the "data speak" about the decision rules the agents are using. Ideally, one would use nonparametric methods for inference. However, as a practical matter, nonparametric rules do not work well when there are more than two or three conditioning variables or the number of observations is moderate in size, as will be the case in our application. As is well known, the curse of dimensionality in these methods generates very poorly estimated models. In our application, we instead use specifications that are parametric, but allow for flexibility through the inclusion of splines and higher order terms when the data are sufficiently rich.

We use an ordered probit for the housing investment policy function. For a household, i, that is a homeowner's ($h_i > 0$), the decision about how much housing to investment in for the period, h'_i, belongs to one of four ordered categories $j \in$:

1. rent: $h'_i = 0$,
2. downgrade: $0 < h'_i < h_i$,
3. remain in existing home: $h'_i = h_i$,
4. upgrade: $h'_i > h_i$.

The theoretical model implies that an agent's policy function in the dynamic program should depend on the state variables. We label the state variables at time t as $\mathbf{s}_{it}$. These include variables such as home prices, housing stock, current income, and assets. Because income follows a life-cycle process, we keep track of individual specific state variables such as age. We include family size as a state variable because it may affect housing preferences.

We specify the index representation of the policy function $h'_i = F(\mathbf{s}_i; \beta) + u_i$ as a flexible function $F(\cdot)$ of the state variables with parameters β. Under the ordered probit model, the probability of household i making a housing investment decision in one of the four ordered categories j at time t is given by

$$p_{it,j} = P[h_{it} \in j] = \Phi(\alpha_j - F(\mathbf{s}_{it}; \beta)) - \Phi(\alpha_{j-1} - F(\mathbf{s}_{it}; \beta)), \tag{6}$$

where $\Phi(\cdot)$ is the standard normal cdf and $a_j, j \in \{1, 2, 3\}$, denote cutoff values separating the four ordered categories. For renters ($h = 0$), we maintain the same ordering, but there are only two possible categories: $j \in \{h'_i = h_i = 0, h'_i > h_i = 0\}$. Although renters could be grouped with homeowners, we estimate a separate ordered probit (equivalent to a binomial probit) for renters to allow for more flexibility. The ordered probit model gives rise to the lumpy investment pattern in housing investment predicted by the theoretical model with adjustment costs.

A convenient feature of the specification is that the preference shocks on housing in the structural model, κ_i, have a one-to-one mapping with the unobserved shocks u_i in the policy functions. The theoretical model predicts that the housing policy function is (weakly) increasing in the preference shocks. Similarly, a monotone relationship between u_i and housing investment holds in the reduced form policy functions. As shown in BBL for models with continuous control variables, the monotone relationship allows us to invert the preference shocks into the policy function shocks and, hence, estimate the parameters describing the distribution of the preference shocks. The mapping is especially convenient because both shocks are normally distributed.

Conditional on a household making a housing stock adjustment, we model the size of the adjustment as a function of the same state variables in the ordered probit model. For flexibility, we allow for an asymmetric adjustment size in upgrades, downgrades, and renters switching to ownership. To guarantee positive adjustments, we use the difference in the log size of adjustment:

$$|\log(h_{it+1}) - \log(h_{it})| = F_{adj}(\mathbf{s}_{it}; \beta_{adj}), \tag{7}$$

where the adjustment, adj, is either an upgrade, downgrade, or renter switching to home ownership.

We have also experimented with more elaborate specifications for the housing investment policy functions. However, the number of adjustments in our data is a small fraction of the overall observations. Also, in our second step estimates, we will need to forecast individual level housing decisions. The time series literature on forecasting tells us that overparameterized models typically generate poorer forecasts than more parsimonious specifications. Therefore, we opt for this simple specification.[11]

Conditional on a household being a renter, we estimate a model to predict rental value as a function of the state variables. Note that rental value is not a dynamic state variable in our model.

Besides housing investment, households make a dynamic decision about the amount of financial assets to be brought into the next period, a_{t+1}. In our theoretical model, households with mortgage debt cannot simultaneously hold nonhousing financial assets. But in the data, they typically carry both a positive mortgage balance and nonzero nonhousing financial assets, so it is necessary to apply policy functions for both types of financial assets.

We assume homeowners with a positive mortgage balance finance using 30-year fixed real interest rate mortgages,[12] from which we calculate the rate at which home equity wealth accumulates using standard financial calculators. When households move, they roll equity from the previous home into the new home and take out a new mortgage at the prevailing interest rate. Thirty-year fixed rate mortgages were the most common mortgage product in use during our sample period before mortgage market innovations such as adjustable rate, interest-only, reverse, and home equity lines of credit were in common use. The PSID data does not indicate the type of mortgage product, nor can it be inferred from the available data, so it is necessary to take a stance on what product is in use.[13] As mentioned in Section 4, the home equity data are consistent with the amortization of 30-year fixed rate mortgages.

We use data on nonhousing wealth to estimate a policy function for nonhousing wealth accumulation as a function of the state variables. Our wealth data is limited, restricted to the years 1984 and 1989, so we impute annualized saving using both years' wealth observation.[14] With only one first difference in wealth, for years common across all households, we cannot include the interest rate as a state variable because it exhibits no variation.

Finally, we calculate nonhousing consumption using the budget constraint.

6.1. *Reduced Form Policy Function Results.* Table A.3 reports results for the ordered probit model of housing investment for homeowners and Table A.4 for renters. We include income; wealth; the amount of current owned housing; the user cost of housing; and demographics, age and family size as state variables. The coefficient signs are as expected. Households invest in more housing as income, wealth, and family size increase and in less housing as the current amount of housing increases. We apply the user cost method to measure the price of home ownership. We use a simple measure: the difference in real interest rates and the expected real rate of home price inflation. This measure captures both the financing costs of mortgage interest payments and the offsetting investment component of housing capital gains. In our application, we assume households have rational expectations and thus measure expected home price inflation using the realized value of contemporaneous home price inflation. As expected households invest in less housing as the user cost increases. The quadratic term on age reflects

[11] We also experimented with an (S,s) inventory model of durable good expenditures as in Attanasio (2000). This model imposes more structure than our specification. With so few adjustments, the (S,s) model did not yield reasonable results, even in restricted models with symmetric (S,s) bands.

[12] In the United States, mortgages are typically fixed at nominal interest rates. We use real interest rates because we do not separately track inflation. The distinction between real and nominal fixed interest rates affects the rate at which equity accumulates over the life of the mortage.

[13] The data indicate the remaining mortgage balance, which would allow us to track the rate of home equity accumulation. But the data lack information on the term length of the mortgage (30 years, 20 years, 15 years, etc.) and the fixed interest rate, which may have been set several years ago in the past. Without these two variables, we cannot infer the type of mortgage product or refinance and prepayment activity.

[14] We construct the total contribution to nonhousing wealth between 1984 and 1989 by taking the difference between wealth in 1989 and what the value of the household's 1984 wealth would have been in 1989 had it accumulated interest at the interest rates recorded in our data. We compute the annual saving using an arithmetic average: dividing the total contribution by 5 years.

the hump-shaped pattern in housing investment over the life cycle. The signs of these effects hold in the models for both current homeowners and renters.

We find down-payment constraints are quite important. For the homeowner model, we include linear splines in the ratio of wealth to home value to capture the effect of down-payment constraints. Homeowners that are above a wealth to house value ratio of 30% are likely not credit constrained, and the results indicate that the marginal effect of additional wealth has little effect on housing decisions. But for households that face constraints, below 30%, the marginal effect is larger. For those households, an incremental drop in wealth is correlated with a much higher probability of moving down the housing ladder. For the renter model, we include linear splines in the ratio of wealth to rental value. For the purpose of comparing results to the homeowner model, we scale rental values into rental house values using an owner's equivalent rental rate of 7.5%. We also use a finer partition in the splines. Above 30%, wealth has almost no effect on the decision to upgrade to home ownership, between 10% and 30% wealth has a large effect, and the effect is moderate at very low-wealth levels below 10%. At very low levels, the effect should be smaller because an additional dollar of wealth will not be enough to cross the threshold of typical down-payment constraints. The down-payment effects are more pronounced for renters than for homeowners. This could in part be attributed to the fact that mortgage contracts do not force existing homeowners to downgrade if the equity ratio dips below the threshold. Although in theory it is desirable to use more flexible nonparametric functional forms for the other state variables besides wealth, in practice, we need to use more parametric functional forms given the low frequency of upgrades, downgrades, and switches to renting.

The cutoff values in the ordered probit give rise to infrequent moves. In particular the wide gap in the cutoff values between upgrades and downgrades indicates that in most periods households remain in their existing home. Indeed, the data indicate that about 90% of households remain in their existing home.

Table A.5 reports the results for the adjustment size models estimated using the subset of households that adjust. We will note that, conditional on the state variables, the adjustments are large and reflect the lumpiness in housing investments. Table A.6 reports the estimates of the house value for renters switching to home ownership. Table A.7 reports the estimates of rental values for renters. Table A.8 reports results for the nonhousing wealth policy function. The results indicate saving increases in income and age, and decreases in total wealth (nonhousing plus home equity), family size, and either annual mortgage payment or rent payments. As noted before, we cannot include the interest rate as a state variable. As a practical matter, we need to use a relatively parametric specification to mitigate the adverse effects of measurement error in our saving variable propagating into the second stage of the estimation procedure.

6.2. *Exogenous State Variables: Income, Home Prices, and Interest Rates.* In BBL, the econometrician must also estimate the stochastic processes governing the law of motion for the state variables. The process for income includes an age component, a cohort effect, household random effects, and an AR(1) error disturbance:

$$\begin{aligned} \log(y_{it}) &= \beta_0 + \beta_1\, age_{it} + \beta_2\, age_{it}^2 + \beta_3\, birthcohort_i + \alpha_i + \epsilon_{it} z_{it}, \\ z_{it} &= \rho z_{it-1} + \nu_{it}, \\ \epsilon_{it} &\sim N(0, \sigma_\epsilon^2), \\ \nu_{it} &\sim N(0, \sigma_\nu^2), \\ \alpha_i &\sim N(0, \sigma_\alpha^2). \end{aligned} \tag{8}$$

Estimates are reported in Table A.9, which shows that income is persistent and exhibits a hump-shaped pattern over the life cycle.

We model the time series process for real interest rates r_t and real home price inflation π_t as a Vector Auto Regression (VAR) with one lag,

$$
\begin{aligned}
r_t &= \beta_{cr} + \beta_{rr} r_{t-1} + \beta_{r\pi}\pi_{t-1} + e_{rt}, \\
\pi_t &= \beta_{c\pi} + \beta_{\pi\pi}\pi_{t-1} + \beta_{\pi r} r_{t-1} + e_{\pi t},
\end{aligned} \tag{9}
$$

where the error term is distributed bivariate normal, $\mathbf{e} \sim \mathbf{N}(\mathbf{0}, \Sigma)$. We use a longer time series that spans the years 1975–2009. Results are presented in Table A.10. The estimates capture significant persistence in both time series, and there is a distinct pattern of "boom and bust" persistence. The process forecasts persistent periods of low interest rates/high home price inflation and periods of high interest rates/low home price inflation.

6.3. *Goodness of Fit.* Using our estimated decision rules and law of motion, it is possible to simulate the entire life cycle of housing decisions for an agent in our model. This simulation is a key input into the Bajari et al. (2007) estimator. We evaluate the goodness of fit of the simulations by tracking the simulated life cycles of the youngest cohorts we use for the estimates.

Table A.11 reports averages of several key variables from the simulations in 5-year intervals for the five youngest cohorts. The table is constructed in a similar way to the cohort Table A.2 in the summary statistics section. It should be noted that these tables are not directly comparable because the simulations track a single cohort over a full life cycle and the cohort table tracks many cohorts for just a 13-year time frame. Nonetheless, comparisons are still useful for evaluating the goodness of fit.

Overall, the fit is sensible. The first point to note is that the initial trajectories of the simulations are quite reasonable; this can be seen by comparing the values in the data to the first 5 or 10 years of forward simulations. Over a longer horizon, the general trends in the simulated life cycles match the trends found in the data. Income displays a distinct hump-shaped pattern. Both wealth and housing (whether owned or rented) increase over the life cycle. They do not have a distinct hump-shaped pattern, but the rate of wealth and housing accumulation diminishes at older ages. Consistent with the data, households leave large amounts of wealth as bequests. We also see the rate of renting decline rapidly at young ages and then level off at middle ages. The various measures of moving frequencies match remarkably well. There seems to be some bias toward home ownership over renting, but we view this favorably because the years following our sample period in 1993 did in fact see a boom in housing markets.

7. SECOND STAGE

In this section, we describe the second stage of the BBL estimation procedure that estimates the structural utility function parameters. For the sake of brevity, we do not explain the entire estimation procedure in this article; the interested reader is referred to the original paper. In the second stage, the econometrician first simulates life-cycle housing and nondurable consumption under the policy functions that we estimated in the first stage. Next, he reverse engineers the preference parameters that make this observed policy "optimal" in the sense that no alternative policies yield higher utility.

We estimate the model on a subsample of 450 households of family size 2. We use this family size for the counterfactual exercises in the next section, and for robustness we compute point estimates for a family size of 4. We simulate 150 life-cycle paths per household. We use 20 alternative policies constructed by perturbing the parameters values in the reduced form policy functions in a range of approximately $\pm 15\%$. Despite the computational advantages of the method, this is the largest sample that our workstation can accommodate. We use 120 bootstrap replications from the subsample to calculate standard errors.[15] Further details are presented in the appendix.

The results are presented in Table 1. Of note, the elasticity of substitution parameter, τ, is significantly greater than zero. The home ownership preference shock parameters are reported

[15] This procedure does not account for first stage error in the construction of standard errors.

TABLE 1
UTILITY PARAMETER ESTIMATES

Parameter	(1)	(2)	(3)
Consumption share θ	0.5390	0.5827	0.6127
	(0.0228)		
Elasticity of sub. τ	0.7802	0.7246	0.6796
	(0.0451)		
Homeownership preference shock (mean) μ_κ	−3.0835	−3.3342	−3.3396
	(0.0622)		
Homeownership preference shock (std. dev.) σ_κ	0.7051	1.3771	1.4740
	(0.1148)		
Bequest γ	2.4412	2.2780	1.3044
	(0.0457)		
Adjustment cost	6%	10%	6%
Family size	2	2	4

NOTES: Standard errors in parentheses, only calculated for preferred specification. The ownership preference shocks are distributed log-normal. Converted to levels, the mean shock is 0.0587 in the first specification. The second and third specifications are estimated on a smaller sample size of 100 households.

for the log-normal distribution. Converted to levels, the mean preference shock is 0.059, which is similar to the owners equivalent rent value of 0.075 that we use as a benchmark for the descriptive evidence.

As a validation of the results, it is useful to take the estimated parameter results and calculate implied housing shares in a static model of housing demand.[16] For homeowners, evaluated at the mean value of the home ownership preference shock, the implied expenditure share on housing is 17.1%, which compares quite favorably to the 17.9% share in the raw data. For renters, the implied housing expenditure share is 32.9%, which is higher than the 20.9% share in the raw data for the average renter. However, this share compares favorably to the data for the low end of the income distribution. Below $10,000 income, the bottom third of the income distribution, the average share in the data is 31.1% for renters. We may be getting this result because our model has difficulty matching the behavior of high-income renters whose rental decisions may be driven by factors other than household financial fundamentals.

As a robustness check, we estimate the model assuming an adjustment cost of 10% as opposed to 6%. We also estimate the model for households of family size 4. Table 1 reports point estimates. Quantitatively, the estimates are quite similar. Assuming higher adjustment costs, the elasticity of the substitution parameter is slightly lower, but it remains much larger than 0. Preferences are also tilted toward home ownership: In levels, the mean preference shock is 0.092 as opposed to 0.059. Preferences are similar for larger families, with the differences being a slightly lower elasticity of substitution and tilt toward home ownership. It is interesting that the bequest parameter is much smaller for larger families.

8. SIMULATION RESULTS FROM THE STRUCTURAL MODEL

8.1. *Parameterization.* In the interest of clarity we summarize the parameters used for the simulations of the structural model in Table 2. Whenever possible and applicable we use the parameters estimated and employed in the previous sections.[17]

We now use these parameters to simulate the response of consumption and asset accumulation choices to income and house price shocks. We proceed in three steps. First, in order to briefly explain the main mechanisms of the structural model, we display the life-cycle profiles in the absence of income, house price, and preference shocks. Then we subject households to joint

[16] In the static model, households maximize the period utility function subject to a budget constraint on housing rental and consumption spending. It is assumed that the housing rental rate is 7.5% of a home's value.

[17] Note that the range of the preference shocks used was determined as $e^\kappa = \{e^{\mu_\kappa - \sigma_\kappa}, e^{\mu_\kappa + \sigma_\kappa}\}/g$, where the scaling by the utility parameter g is necessary since in the empirical estimation in the previous section g was normalized to 1.

TABLE 2
PARAMETERIZATION OF STRUCTURAL MODEL

Parameter	Value	Interpretation
	Preferences	
β	0.97	Time discount factor
γ	2.56	Degree of altruism
σ	1	Intertemporal elasticity of subst.
g	7.24%	Service flow from housing stock
θ	0.539	Consumption share in utility function
τ	0.7802	Elasticity of sub. between c, h: $\frac{1}{1-\tau}$
e^{κ}	$\{0.31, 1.28\}$	Domain of pref. shocks
	Housing and Financial Markets	
ϕ	6%	Transaction cost
ξ	20%	Down-payment requirement
r	1%	Return on financial assets
r_m	7.24%	Mortgage interest rate
ρ	7.24%	Rental rate of housing
π_p	0.95	Persistence of house price shock
σ_p	0.1	Std. dev. of house price shock
	Labor income process	
π_η	0.95	Persistence of income shock
σ_η	0.3	Std. dev. of income shock
ε_t	Hansen (1991)	Life-cycle labor income profile

income and house price shocks, under the benchmark parameterization. Finally, we assess the importance of down-payment constraints, adjustment costs, and the elasticity of substitution between consumption and housing services in the utility function for household responses to these shocks. The first sensitivity analysis alters the down-payment constraint, which is motivated by the recent financial and macroeconomic crisis that has led to changes in the extent to which households can borrow against the value of their home. The second exercise shows the importance of adjustment costs by setting it to zero. The third exercise highlights that precise estimates of structural utility parameters are quantitatively important in assessing the model-implied consumption and wealth response to income and house price shocks.

8.2. *The Mechanics of the Model.* Figure 1 displays life-cycle profiles of income shocks,[18] η_t, and house prices, p_t (the exogenous stochastic driving forces of the model[19]), consumption of the composite good, c_t, financial assets, a_{t+1}, the housing choice, $o_t h_{t+1} + (1 - o_t) l_t$, and a variable we call voluntary equity, $q_{t+1} = a_{t+1} + (1 - \xi) p_t h_{t+1}$.

As explained in the appendix, it is helpful computationally to use this variable instead of financial wealth in the recursive formulation of the household problem. However, introducing this variable is not only useful for computation, but also helps to interpret the simulation results. Note from the financing constraint (3) it follows that $q_{t+1} \geq 0$ and $q_{t+1} = 0$ if the down-payment constraint binds. The variable q_{t+1} measures the equity stake in housing, in excess of the fraction $1 - \xi$ required by the financing constraint. Thus $q_{t+1} = 0$ indicates that in period t households are financing-constrained whereas $q_{t+1} > 0$ indicates a nonbinding constraint. For the figure (as for all simulations to follow) households start with zero financial and housing assets.[20]

[18] Recall that total labor income is given by $y_t = \eta_t \varepsilon_t$. Thus what we plot here is income net of the deterministic life-cycle component, which makes the income *shocks* the household faces more clearly visible. However, in the presence of financing constraints the hump-shaped average income profile $\{\eta_t\}$ is clearly important for the dynamics of consumption and wealth over the life cycle plotted in the figures, especially in the early stages of economic life.

[19] The final exogenous driving process of household choices is the preference shock. Unless otherwise noted we simulate households with constantly high realizations of the preference shock for owner occupied housing, and thus do not plot this (constant) time series in the figures.

[20] Recall that households can rent and adjust their stock of owner occupied housing at the beginning of the period. Thus zero-initial housing wealth does not imply that the consumption of housing services is zero in the first period of economic life.

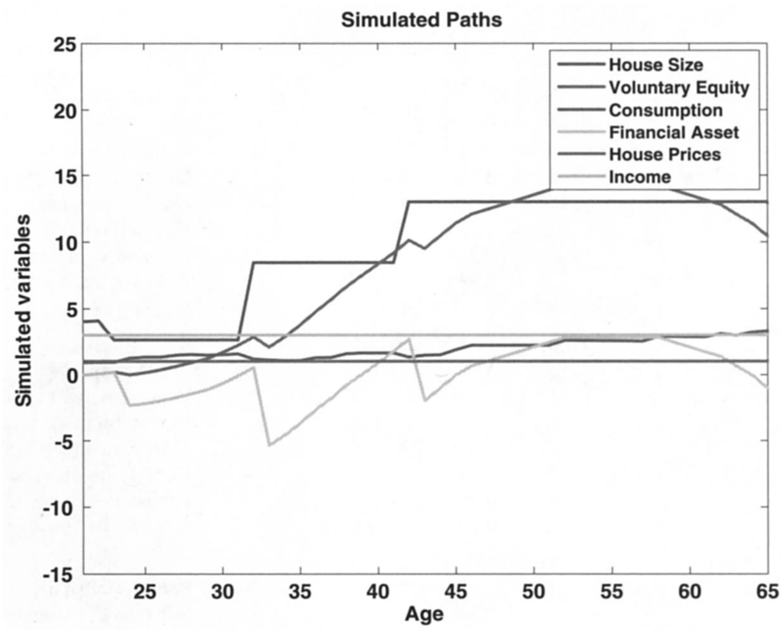

FIGURE 1

LIFE-CYCLE PROFILES IN THE ABSENCE OF SHOCKS

Underlying the life-cycle paths are, of course, the decision rules in the presence of income, house price, and preference shocks, but for the benchmark simulations the *realizations* of the income and house prices as well as preference shocks are constant sequences. Concretely, here we assume that households face high house prices and receive good income realizations and preference shocks that favor owner occupied housing.

From the figure one can clearly observe the key housing market frictions in action. Early in the life cycle the household decides to rent, despite the fact that she has a fairly strong preference (due to the realization of the preference shock) for owning. This is due to the fact that because of the down-payment constraint and low-labor income early in the life cycle the household could not have bought a home of reasonable size. The first home the household buys (at the age of 23, the third model period) is actually smaller than the previously rented home because she cannot finance a larger purchase in the presence of the collateral constraint. Note that in the first period of home ownership voluntary equity falls to zero, that is, the household has taken out a mortgage with the maximum permitted leverage of 80%. A larger home purchase at this stage of the life cycle would only have been possible by further compromising nondurable consumption, which is already low relative to the life-cycle consumption profile a financially unconstrained household would choose.[21]

The household remains a home owner for her remaining life (again this is true only for the assumed realizations of the preference shock).[22] Due to the nonconvex adjustment cost in owner occupied housing she alters the size of her house infrequently. Owner occupiers move three times during the first 20 years of their lives, reaching their desired housing size at the age of 42 and then, absent further shocks to house prices and incomes, stay put. The fact that households move several times prior to reaching their unconstrained optimal size of the house is again due to the presence of the financing constraint. This is most apparent for the first two

[21] Note that reducing nondurable consumption is not overly costly in utility terms, given the estimated high elasticity of substitution between nondurable consumption and housing services.

[22] For realizations of preference shocks that induce households to rent, the resulting life-cycle paths mimic those of the frictionless model discussed below because renters do not face housing adjustment costs in the model.

moves, because in the presence of the down-payment constraint, the household could not have afforded to finance the house she actually purchased at the age of 30 or 23, the age at which the first home was bought. The 20% down payment on that house would have exceeded her disposable income. Matters are different at the time of the second move as owner occupier, at the age of 30. Now the household could have afforded a larger house. Note that voluntary equity drops at the time of the home purchase, but not all the way to zero, indicating that the collateral constraint is not binding after the housing transaction. In fact, the household could have managed to buy the larger house actually purchased at the age of 42, but only by leveraging up to the maximum and by significantly reducing nondurable consumption. This is not optimal at the age of 30; instead the household first accumulates further voluntary equity (pays back mortgage debt) before purchasing the larger (and final) home.[23]

We conclude that in order to reproduce the empirical facts that households adjust the size of their home infrequently, but more than once on average over their lifetime, the combination of both frictions in the housing market is crucial. Quantitatively the model reproduces the average time in between housing adjustments and the number of moves during a households' lifetime documented above for U.S. data rather well, at least for the early part of the life cycle.[24]

Having discussed how households behave in the absence of income and house price shocks, we are now prepared to explain how these households, within the model, respond to simultaneous declines in income and house prices as observed recently for the U.S. economy.

8.3. *Simulating an Income and House Price Shock.* The exercise we carry out is intended to mimic a sudden, unlikely, but not entirely inconceivable (from the households' perspective) decline in the price of housing. At the same time the household receives a negative income shock (which by construction is highly but not perfectly persistent).[25] Given the stylized nature of our structural model, which is necessitated by our desire to estimate it and provide a reasonably tight link between theory and estimation, we view our precise quantitative results as less important relative to exhibiting to what extent model elements and parameters (e.g., adjustment costs, the size of the down-payment constraint) affect household responses.

In Figure 2, we display the life-cycle patterns of consumption, housing, and financial wealth prior to and following a negative house price shock. We show the same variables, but following an income shock, in Figure 3 and plot the effect of the concurrent event of both shocks in Figure 4. The income and house price shocks are assumed to hit early in a household's life, prior to the age of 42 when the household, in the absence of shocks, would have acquired its optimal-housing size (see Figure 1).

A reduction in house prices alone does not alter the basic dynamics of life-cycle consumption, housing, and portfolio choice, but it does affect the timing as well as the size of the adjustment in the owner occupied home. Concretely, in response to the fall in real estate prices, households purchase their third and final home more quickly (3 years earlier), and the acquired house is about 30% larger than the one they would have bought in the absence of house price declines. Interestingly, and again due to the presence of the financing constraint, households do not purchase the larger home immediately following the price drop, but rather accumulate financial wealth for another 2 years prior to making their large real estate transaction. As before, they could have bought a home this size immediately following the house price fall, but only by

[23] Because there is a chance that house prices will fall in the future, at the age of 30 there is an option value of waiting to purchase a larger house. In addition, in the presence of income shocks, driving voluntary equity to zero is suboptimal because it prevents the household from smoothing future income shocks (short of selling the house and incurring the transaction costs of purchasing a smaller one).

[24] To the extent that the model abstracts from housing transactions triggered by relocation shocks we would expect the model to understate the frequency with which households move. This is apparent for households in the later stages in their life cycle for which the model, absent income and house price shocks, predicts no housing size adjustments at all.

[25] Of course, because we use the decision rule of households derived under the Markov process for income and prices, households at any period understand that the state of the Markov process can switch with certain probability. We simply trace out the dynamic response of households to a particular *realization* of the stochastic process.

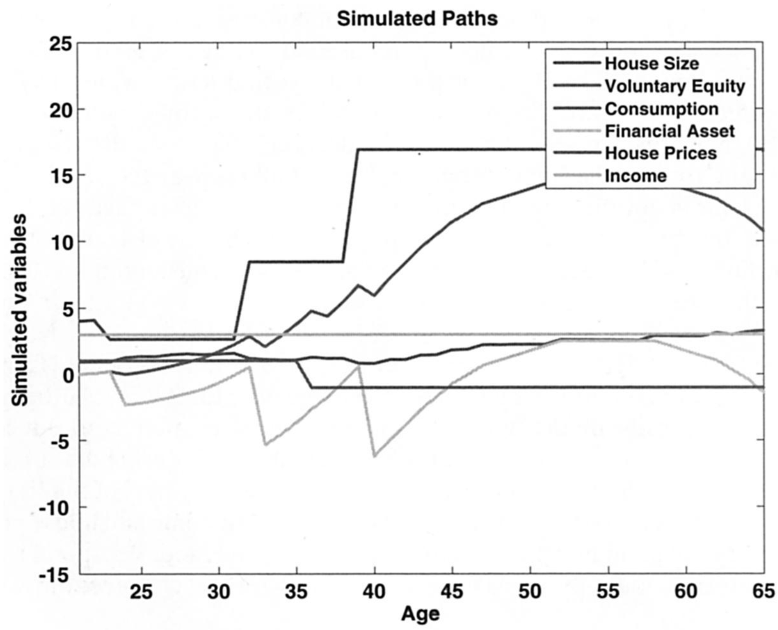

FIGURE 2

THE EFFECTS OF A HOUSE PRICE DECLINE

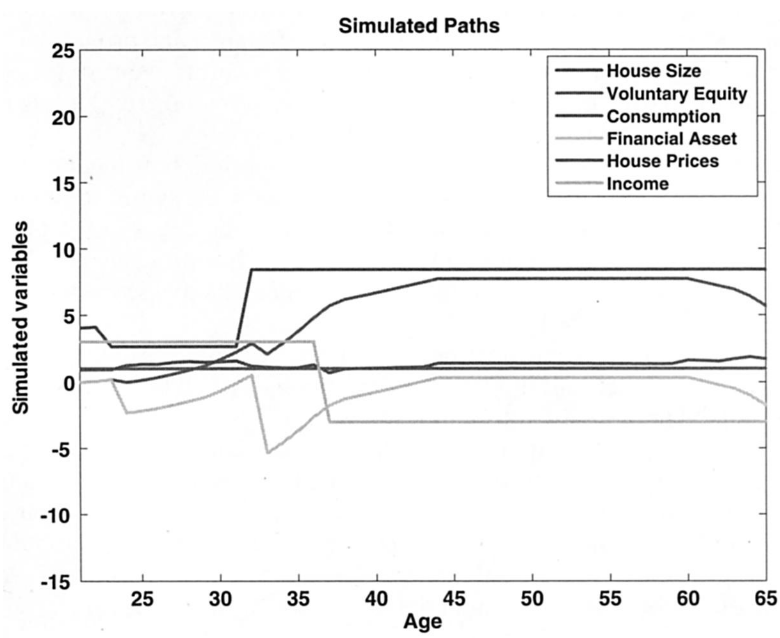

FIGURE 3

THE EFFECTS OF A DECLINE IN LABOR INCOME

letting nondurable consumption fall and/or driving mortgage debt to the maximum permissible level.[26]

[26] Note that house prices are very persistent, and thus the risk of a reversal in these prices prior to the household purchasing her larger home is small.

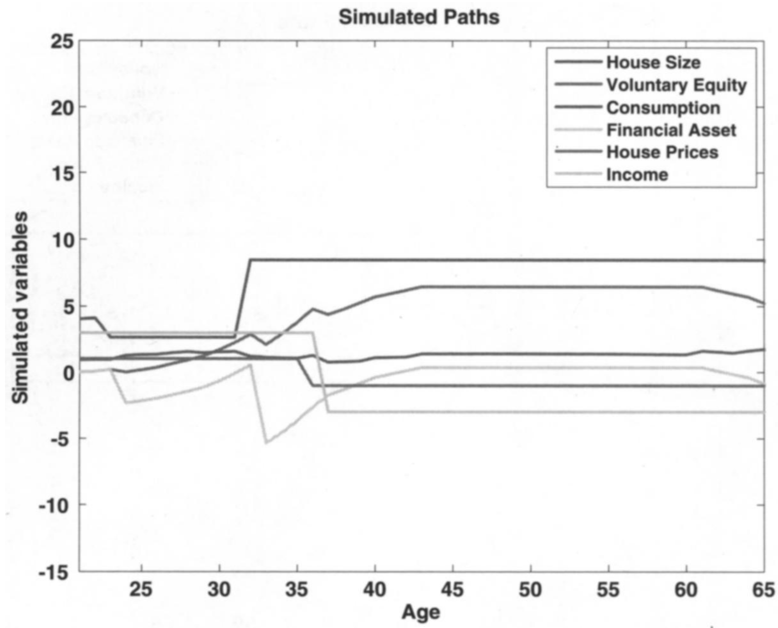

FIGURE 4

THE EFFECTS OF A SIMULTANEOUS DECLINE IN LABOR INCOME AND HOUSE PRICES

As Figure 3 suggests, starting from a high income level, an adverse labor income shock *does* have a qualitative impact on the desired life-cycle profile of housing choices. Now the household abstains from purchasing a larger home during her middle ages and, absent a recovery in her labor income, remains in the smaller home for the rest of her life. Note that this would not have been the desired size of the home of this household had income always been low.[27] This thought experiment demonstrates that a fall in labor income induces a decline in the demand for owner occupied real estate and that this fall will in part come from middle aged households that would have upsized their homes in the absence of adverse income shocks. As expected from the standard permanent income hypothesis, nondurable consumption also bears a downward adjustment in response to the income loss (which is expected to be very persistent).

Finally, Figure 4 simulates the effects of a simultaneous collapse in labor incomes and house prices, our stylized description of the recent crisis in the housing and labor market. It shows that the decline in income "dominates" in the sense that households, relative to the no-crisis scenario, continue to postpone home purchases due to the fall in their labor incomes, despite the fact that the price of real estate has become significantly cheaper. Figure 4 is substantially identical to Figure 3, which stands in sharp contrast to the consequences of a fall only in house prices depicted in Figure 2. Had only real estate prices declined without a loss of labor income, at least middle aged households with some housing equity would have continued to want to trade up, in fact, trade up more and earlier because of the more affordable homes. A collapse of labor income makes this suboptimal[28] and thus generates adverse consequences for the demand for real estate.

[27] Of course, given that income is driven by a mean-reverting Markov chain, households perceive a positive probability that income will eventually recover. As long as it does not, the household in fact prefers to rent despite the fact that, based on the realization of the preference shock, she would prefer to own, other things equal.

[28] Note that it does not make it infeasible, at least for the middle-aged households discussed here. These households have sufficient funds to trade up to the house size as in Figure 2, but with their lower labor income would have to compromise more in nondurable consumption to service the associated mortgage. That they find suboptimal.

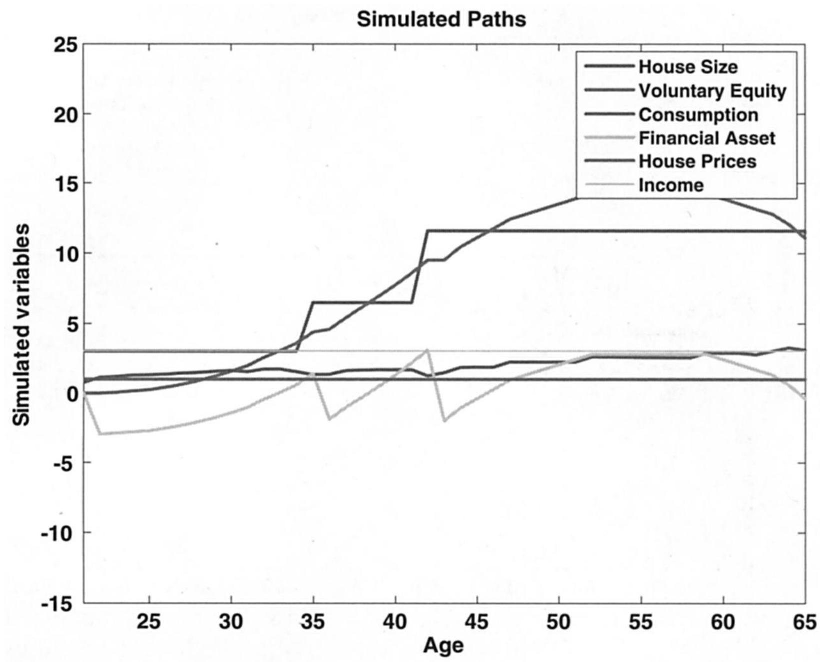

FIGURE 5

LIFE-CYCLE PROFILE WITH RELAXED DOWN-PAYMENT CONSTRAINT

9. SENSITIVITY ANALYSIS

In this section we perform a sensitivity analysis with respect to three parameters. The first set of comparative statics results is motivated by recent policy-relevant changes in the U.S. mortgage market. In particular, we want to deduce the impact, within our model, of a tightening of credit lines, as observed in the current crisis of the U.S. mortgage market.

Second, we demonstrate that the model with nonconvex adjustment costs on housing gives fundamentally different predictions about household responses to income and house price changes than the frictionless benchmark model of consumer durables (as put forward by Mankiw, 1982) in which the adjustment of the stock of housing is completely costless.

Finally, we have spent considerable effort in precisely estimating the preference parameters of our model, in particular the elasticity of substitution between nondurable consumption and housing services in the utility function. We therefore want to investigate to what extent the results of our model depend on this parameter. To this extent we repeat our simulations with a Cobb–Douglas utility specification that is commonly employed in macroeconomics (see, e.g., Fernandez-Villaverde and Krueger, 2011, or the discussion in Jeske, 2005).

9.1. *Relaxing the Financing Constraint.* Our model is rich enough to address the question, albeit in stylized form, of what happens to household's housing and nondurable consumption decisions as the financial sector tightens credit lines for mortgages. We now draw out the household response to a simultaneous decline of income and house prices under the assumption that households are required to hold only a $\xi = 10\%$ equity share of the value of their home as opposed to $\xi = 20\%$ as modeled so far. One can interpret our benchmark scenario with tightened credit as the situation after the crisis hit and the relaxed constraint scenario as the situation prior to the current financial crisis.

Comparing Figure 5 to our benchmark results in Figure 1, we observe that an important consequence of a relaxed down-payment constraint is that households now purchase their own home right away. Again, recall that throughout these simulations we endow households with a (realized) preference for owning. Nevertheless, under the tighter down-payment constraint in Figure 1 the household preferred to rent instead of being forced to live in a suboptimally small owned home early in the life cycle. With the relaxed constraint, households buy immediately, and thus do not transit from being renters to home owners later in life. The first house they purchase is somewhat larger than the one households buy under a tighter collateral constraint. The remaining life-cycle profile is similar to that of the benchmark. This similarity is not surprising because in the benchmark parameterization with the tighter down-payment requirement, the constraint ceases to bind shortly after the initial transition to home ownership; see again Figure 1.[29] We conclude that the main impact from a tightening of the down-payment constraint is to delay demand for owner occupied housing by young households and to reduce the size of the first purchase somewhat. Optimal choice of middle-aged households who already have accumulated substantial voluntary equity in their homes, in contrast, is not significantly affected by a tightened collateral constraint.

9.2. *The Role of Adjustment Costs.* The second key friction in the housing market that we model, besides the down-payment constraint, is the presence of sizable transaction costs that need to be borne if (and only if) households change the size of their home. The benchmark model of consumer durables in macroeconomics abstracts from fixed adjustment costs in the market for consumer durables (see, e.g., Mankiw, 1982). Our model nests this specification; by setting the adjustment cost parameter to $\phi = 0$, we obtain the frictionless model.[30] Note that in our model households have access to frictionless rental markets, and thus the adjustment costs only play a role conditional on households deciding to own.

Absent the adjustment costs, the difference between renting and owning is small because an owner occupier can, at no cost, purchase the home, live in it for one period (i.e., rent it to herself), and then sell it back in the next period.[31] Thus, for Figure 6, we assume that households face sufficiently large preference shocks for owning that the near indifference between renting and owning is broken in favor of the latter.

From the figure we observe that the absence of nonconvex adjustment costs changes the life-cycle dynamics of housing choices fundamentally. The lumpy and infrequent adjustment of the housing stock is lost. Rather, the build-up of owner occupied housing in the early stages of a household's life proceeds more gradually.[32] The model without nonconvex adjustment costs also differs substantially from our benchmark in the way households respond to income and house price shocks. Now the stock of housing reacts immediately and significantly to the persistent income decline.

9.3. *The Role of the Elasticity of Substitution.* Finally, in this subsection we show that the dynamic consumption and asset choices depend on the key preference parameters that we estimated in previous sections as quantitatively significant. Specifically, we now document that the optimal choices of households depend on how substitutable nondurable consumption and housing services are in the utility function. Recall that our period utility function was specified as

$$u(c, d) = \frac{1}{\tau} \log[\theta c^{\tau} + (1 - \theta)(\kappa h)^{\tau}],$$

[29] This statement is not completely correct, as the presence of the constraint, even though it is not currently binding, still affects precautionary saving behavior and thus financial asset accumulation and consumption.

[30] Mankiw's model of a representative consumer did not explicitly include down-payment constraints, nor did he calibrate the household income process to micro data.

[31] Owners bear house price risk, though, and, especially early in life, conditional on wanting to own (because of a preference for owing), will face binding collateral constraints.

[32] We solve this version of the model with the same discrete state space dynamic programming techniques as the benchmark in order to allow for the results to be as comparable as possible.

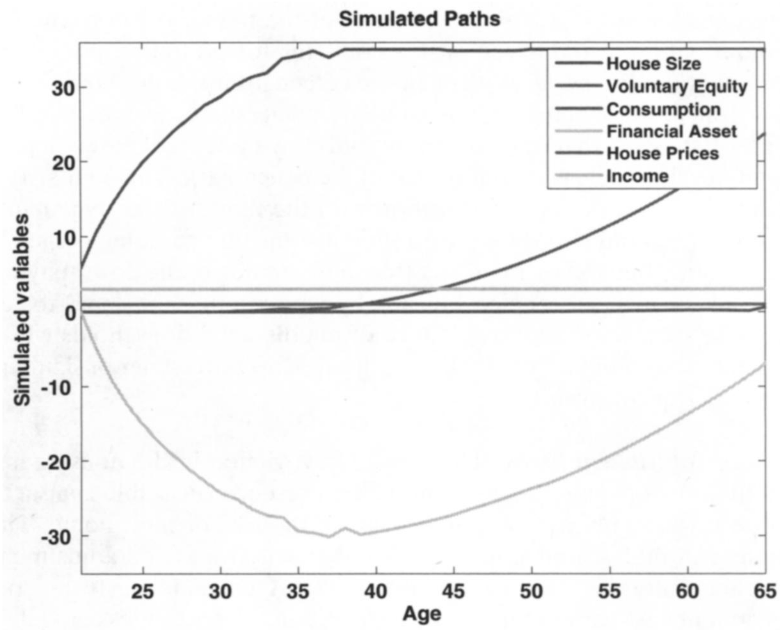

FIGURE 6

LIFE-CYCLE PROFILE IN THE ABSENCE OF ADJUSTMENT COSTS

The elasticity of substitution between nondurable consumption and housing services is given by $\epsilon = \frac{1}{1-\tau}$ and was estimated as $\epsilon = \frac{1}{1-0.7802} = 4.55$. Instead of going to the extremes of perfect or no substitutability (both of which are highly implausible given our empirical point estimates and the small standard errors around these estimates), we document how our result changes if one adopts the familiar Cobb–Douglas specification[33] with unit elasticity of substitution; that is $\tau = 0$ and thus $\epsilon = 1$. Conditional on our choice of a unit intertemporal elasticity of substitution $\sigma = 1$, this case has the additional appeal that the utility function becomes separable in nondurable consumption and housing services. That is,

$$u(c, d) = \theta \log(c) + (1 - \theta) \log(\kappa d).$$

Figure 7, which displays the variables of interest in the absence of housing and income shocks, shows that, although qualitatively, the life-cycle pattern of consumption and housing responses are similar to that in the benchmark case, magnitudes and timing differ markedly. The lower the elasticity of substitution, the more costly it is to sustain a suboptimally low or high housing/nondurables ratio. Note that in our model households can use the frictionless housing rental market to circumvent the frictions generated by collateral constraints and nonconvex adjustment costs in the market for owner occupied homes. Comparing Figure 7 to its counterpart with higher elasticity of substitution (ES), Figure 1, we observe that with lower ES households use the rental market for longer (until age 25 compared to age 23) even though they face preference shocks that favor owning. The first owned home is larger now because households take more time to accumulate wealth, and the first move to a larger house also takes longer now (age 34 as opposed to age 32), as households accumulate more voluntary equity in the old house prior to making this move. Overall, we conclude from the comparison of Figures 7 and 1 that, in the presence of frictionless rental markets, a lower elasticity of substitution between nondurable consumption and housing services leads to a delay in home ownership and upward adjustment of

[33] This parameterization was, among others, adopted by Fernandez-Villaverde and Krueger (2011).

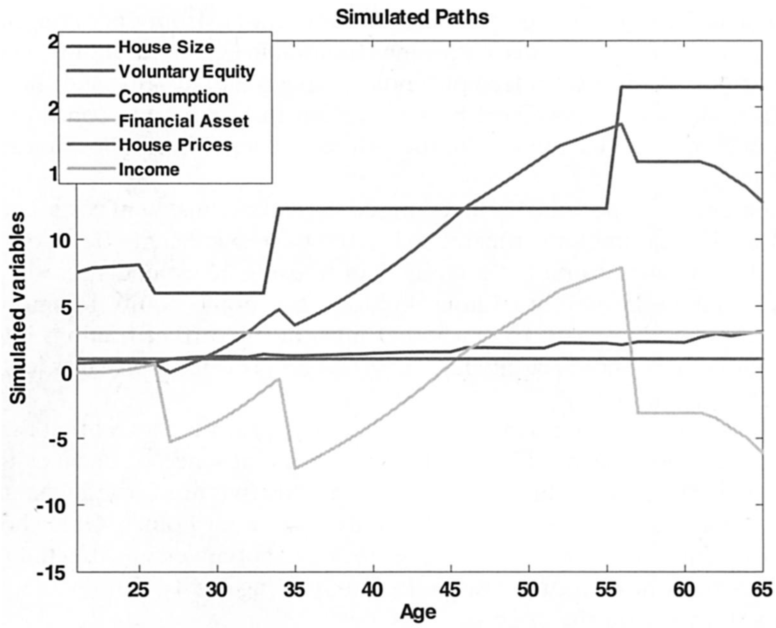

FIGURE 7

LIFE-CYCLE PROFILE WITH COBB–DOUGLAS UTILITY

house size early in the life cycle.[34] The life-cycle profile of nondurable consumption is smoother now as well.

The household response to a negative income and house price shock (not displayed here) is qualitatively[35] similar as in the benchmark economy with higher ES: Households abstain from trading up due to the adverse income shock and stay put in the home they purchased prior to the shock. Financial wealth and nondurable consumption bear the adjustment to the decline in labor earnings.

10. CONCLUSION

In this article, we construct and estimate a dynamic structural model of consumption and housing demand with a frictional housing market. We use our estimated model to simulate counterfactual household responses to a set of negative shocks to income, housing prices, and credit constraints that mimic, in a stylized way, the recent shocks to hit the U.S. economy. In our model, we include two key frictions: down-payment constraints and nonconvex costs to adjust housing stocks. Using household level data from the PSID, we estimate the structural parameters of the model using the two-step method in Bajari et al. (2007).

Because of the two frictions, nonconvex adjustment costs and financing constraints, households find it optimal to make infrequent housing stock adjustments. For the benchmark parameterization, households initially rent while saving for a down payment, then move two times as they the climb the housing ladder before reaching their optimal sized home in the middle stages of their life cycle, at the age of 42. Negative home price shocks allow these households to upgrade earlier in life and to invest in more housing. In this sense, the substitution effect outweighs the income effect of reduced home prices. However, negative income shocks that occur

[34] This finding is qualitatively different from that obtained in a model without rental markets. In such a model the only option the household has to bring up the housing services to nondurable consumption ratio is to purchase a home early in life instead of postponing buying and renting in the meantime.

[35] And, accounting for the different initial condition prior to the shock, the quantitative responses are comparable, too.

at the same time as the house price shock prevent households from upgrading. In response to both shocks, housing demand is lower for young and middle aged households. The age at which the shocks hit is important. For older households, having already reached the optimal home size, the negative shocks are absorbed by a reduction in nondurable consumption, financial wealth, and home equity. The shocks do not induce a change in the housing stock for these older households.

We also document the importance of financing constraints, adjustment costs, and the precision of our utility function parameter estimates. All three have significant effects on the frequency of housing stock adjustments and the lumpiness of housing investments. At the center of this adjustment behavior is the ability of households to use home equity to smooth temporary shocks. Future research has to uncover whether introducing further frictions into the housing finance decision that make home equity lines of credit and reverse mortgages less attractive are able to overturn these results.

Taking a collapse in house prices in isolation, our model results suggest that there is a limited adverse direct effect on housing demand. Rather, in the absence of declines in income, the lower price of real estate will induce younger households that are down payment constrained to move up the housing ladder quicker and to purchase larger homes. Older households that do not intend to further increase their housing stock are not adversely affected (at least in the short run) because they have accumulated sufficient housing equity to not violate the financing constraint even at lower housing prices.

In contrast, the impact from the collapse of real economic activity and the associated declines in labor incomes on housing demand is significantly negative. Although households do not downgrade and sell their homes in the short run, they abstain from purchasing larger homes that they would have otherwise acquired in the absence of the adverse income shock. To more fully assess the housing market details of the great recession, it would be interesting examine foreclosure decisions, subprime mortgage products, and the response of middle aged or old households at or near the borrowing constraint. We defer this to future work.

APPENDIX

A.1. *Computation*

A.1.1. *Recursive formulation of the problem.* The model in recursive formulation can be written as

$$\begin{aligned} V(\eta, a, h, p, t) &= \max_{c,a',h'}\{u(c, g(h)) + \beta EV(\eta', a', h', p', t+1)\} \\ &\text{s.t.} \\ c &\geq 0 \\ h' &\geq 0 \\ a' &\geq -(1-\xi)ph' \\ c + a' + ph' + p\phi(h', h) &= \eta\varepsilon_t + (1 + r(a))a + ph. \end{aligned}$$

A.1.2. *Transformation of the state space.* One problem with the formulation of the problem above is that the constraint set for (a', h') is not rectangular, and that the constraint on h' depends on a', which is itself a choice variable. This problem can be overcome by defining a new variable, voluntary equity, q', as

$$q' = a' + (1-\xi)ph'$$

(see Diaz and Luengo-Prado, 2008; Diaz and Luengo-Prado, 2010). Note that this definition implies that

$$q = a + (1 - \xi)p_{-1}h,$$

where p_{-1} is the price of housing in the previous period, which now becomes an additional state variable. The recursive problem of the household with this transformation of variables now reads as

$$\begin{aligned} v(\eta, q, h, p_{-1}, p, t) &= \max_{c,q',h'}\{u(c, g(d)) + \beta E v(\eta', q', h', p, p', t+1)\} \\ &\text{s.t.} \\ c', q', h' &\geq 0 \\ c + q' + p\xi h' + p\phi(h', h) &= \eta\varepsilon_t + (1 + r(q, h, p_{-1}))q \\ &\quad + [(1-\delta)p - (1 + r(q, h, p_{-1}))(1-\xi)p_{-1}]h, \end{aligned}$$

that is, we traded off an additional state variable p_{-1} against now having a rectangular constraint set for the choice variables (c, q', h'). This is the recursive formulation of the model we compute. Clearly the consistency condition $p'_{-1} = p$ has to hold; that is, tomorrow's past housing price p'_{-1} has to equal today's price. The interest rate function now reads as

$$r(q, h, p_{-1}) = \begin{cases} r & \text{if } q - (1-\xi)p_{-1}h \geq 0 \\ r^m & \text{if } q - (1-\xi)p_{-1}h < 0. \end{cases}$$

Since we use an adjustment cost that is nonconvex, the household decision problem is not a convex programming problem, and numerical approaches that require differentiability of the value function cannot be applied. Therefore we use discrete state space dynamic programming techniques to solve the problem. In particular, we discretize the state space for (q, h) into a finite (but not evenly spaced) rectangular grid (the income and house price process is already a finite state Markov chain by assumption) and maximize the objective function by searching for each (q, h) over the finite grid of admissible choices (q', h'). The consumption choice is implied by the budget constraint.

Given a terminal value function (given by the bequest function), we can iterate backward in age of the household t to solve for the age-dependent optimal policy and value functions. Once we have computed these, simulated life-cycle patterns of consumption, housing, and financial wealth can be generated for any sequence of house price and income shock realizations.

A.2. *Estimation.* In this section we describe the computational details of the BBL estimation method related to forward simulating under the estimated optimal and alternative policy functions, constructing alternative policy functions and estimating the second stage. We only describe the computational mechanics in the appendix. The body of the text discusses the reasoning behind our estimation choices.

A.2.1. *Exogenous state variables.*

Income process. We estimate the income process using a random effects model with an AR(1) error term. See Equation (8). However, for the purposes of forward simulating, we transform the random effect parameter into an imputed fixed effect for each household-year observation. We fit a household fixed effect α_i as

$$\alpha_i = \log(y_{it}) - \left(\beta_0 + \beta_1\, age_{it} + \beta_2\, age_{it}^2 + \beta_3\, birthcohort_i\right).$$

We do not fit the fixed effect using all years for which we observe the household, only for the particular year. That is, for a given household i, the fitted fixed effect α_i could differ for observations corresponding to different years t and t'. The initial income in year 1 of the

simulation is set to the income in the data. The lagged error term in year one is $z_{i1} = 0$. Thus according to Equation (8) the second year error term is $z_{i2} = \nu_{i2}$.
VAR: home prices and interest rates. We set the initial conditions in year 1 to the values in the data corresponding to the observation year. We simulate forward according the VAR process in Equation (9).

A.2.2. *Optimal-Policy Function Forward Simulations*
Initial conditions. In year 1, we set the variables (rent/own indicator, home size, rental value, remaining mortgage principal, and nonhousing financial assets) to their values in the data. In year 1, all homeowners finance their remaining mortgage principal with a new 30-year fixed rate mortgage fixed at the year 1 interest rate. Households that have a zero balance on their remaining mortgage principal do not have a mortgage. Note again that the mortgage interest rates are fixed at the real interest rate, not a nominal interest rate. Home equity in year 1 is calculated as the difference between the house value and the remaining mortgage principal.
Years: $2 \leq t \leq T$
Timing. Within a given year the timing follows:

1. Update exogenous state variables (income, home prices, interest rates). Update home equity due to home price changes and nonhousing wealth due to interest accumulation.
2. Households make housing investment decision and renting decision.
3. If household moves to new owner occupied home, originate new mortgage.
4. Make mortgage payments, update remaining mortgage balance, make nonhousing saving decision, and compute consumption.
5. In the final year $t = T$ calculate bequest based on home equity and nonhousing wealth.

Moving decisions. Households make housing investment decisions according to the ordered probit model in Equation (6) and make an adjustment size according to Equation (7). Housing investment shocks are drawn i.i.d. Normal (across households and years). For households choosing to rent, the rental value is determined according to the model depicted in Table A.7. The simulated shocks do not affect the adjustment size or rental value.
Mortgage origination. If the household moves to owner occupied housing, it originates a new 30-year fixed interest rate mortgage locked at the current period real interest rate. It pays a transaction cost equal to 6% of the value of the new home. It rolls the home equity q_{it} from the previous home into the new home. For homeowners switching to a new owner occupied home the principal on the new mortgage is calculated as

$$Pr_{it}^{mortgage,0} = p_{it}h_{it}^{new} + 0.06p_{it}h_{it}^{new} - q_{it}.$$

If the principal is calculated as a negative amount (as could happen if the household downgrades), the household does not originate a mortgage and the excess balance is placed in nonhousing wealth. We do not directly impose a down-payment constraint for homeowners switching to a new owner occupied home. For renters switching to ownership, the principal on the new mortgage is calculated as

$$Pr_{it}^{mortgage,0} = 0.8 * (p_{it}h_{it}^{new} + 0.06p_{it}h_{it}^{new}).$$

There is a 20% down payment that is deducted from nonhousing wealth. It is possible for this deduction to result in negative nonhousing wealth and/or a negative total wealth position. If the household does not move to a new owner occupied home, it does not originate a new mortgage.

Mortgage calculations. We calculate mortgage payments and update the remaining mortgage principal using standard mortgage formulas with annually compounding interest (as opposed to daily or continuous compounding). The calculations are based on the principal and interest rate in the year in which the mortgage was originated. All households with a mortgage, including those that just moved, make a mortgage payment. After adjusting the remaining mortgage balance we update home equity due to the payoff of principal.

Nonhousing saving decision. Households make a nonhousing saving decision according to the model depicted in Table A.8. The simulated shocks do not affect the saving decision.

Consumption. We calculate consumption using the budget constraint and impose a \$1000 subsistence requirement. For owners,

$$c_{it} = \max\left\{y_{it} - Savings_{it} - Pay_{it}^{mortgage}, 1000\right\}.$$

For renters,

$$c_{it} = \max\left\{y_{it} - Savings_{it} - Pay_{it}^{rent}, 1000\right\}.$$

Bequest. At the end of the last year of the life cycle in year $t = T$, households make a bequest b_{iT} equal to the sum of home equity and nonhousing wealth, subject to a \$100 minimum bequest:

$$b_{iT} = \max\{q_{iT}, 100\}.$$

A.2.3. *Alternative policy functions.* We forward simulate and calculate expected discounted utility for each household under 20 alternative policy functions by perturbing parameters in the policy functions for housing investment, adjustment size, non-housing savings, and rental value. The alternatives are drawn to be different for each household. To identify the preference shock parameter, we hold fixed the housing investment shocks in each alternative. The perturbations are drawn independently across parameters from a normal distribution, with the variance scaled to 15% of the parameter's value. We use a 5% scaling on the savings policy function parameters. In addition to a component that independently perturbs each threshold in the ordered model, we also use perturbations that shift all of the thresholds together.

A.2.4. *Second stage.* We randomly select with replacement a sample of 450 observations. We minimize the objective function as described in Bajari et al. (2007) using Matlab's simplex based minimization routine *fminsearch*. We experimented with alternative starting values and always converged to the same estimates. On an earlier and slightly different variant of the model, we validated the minimization routine using a grid search. We use 120 bootstraps to compute standard errors. We do not bootstrap the first stage, and thus our second stage standard errors do not account for first stage error.

A.3. *Tables*

TABLE A.1
SUMMARY STATISTICS

Variable	Num. Obs.	Mean	Std. Dev.	Min.	Max.
		All Households			
Income	45,533	23,479	16,072	2000	119,450
Family size	45,533	3.08	1.55	1	15
Age	45,533	42.90	12.75	25	70
Renter indicator	45,533	0.323	0.467	0	1
House value	45,533	45,619	39,071	1543	472,339

(*Continued*)

TABLE A.1
CONTINUED.

Variable	Num Obs	Mean	Std Dev	Min	Max
	All Households				
House value/income	45,533	2.388	2.123	0.038	76.481
Housing services	45,533	3421	2930	116	35,425
Housing services/income	45,533	0.179	0.159	0.003	5.736
Annual payment (rent or mortgage)	45,533	2251	2469	0	57,193
Total wealth	45,533	49,116	73,342	−23,684	707,099
Nonhousing wealth	45,533	26,644	52,118	−9945	396,586
Home equity wealth	45,533	22,472	33,270	−15,728	436,264
Total wealth/house value	45,533	1.067	2.086	−2.006	148.600
Total wealth/income	45,533	2.051	3.283	−3.902	84.152
Annual nonhousing savings	2108	103	11,386	−83,137	62,023
Move indicator (inc. rent-to-rent)	45,265	0.147	0.354	0	1
Move indicator (exc. rent-to-rent)	45,265	0.068	0.252	0	1
	Renters				
Income	14,737	14,936	10,789	2000	118,969
Family size	14,737	2.77	1.69	1	13
Age	14,737	38.96	12.55	25	70
House value	14,737	30,647	18,176	1674	289,605
House value/income	14,737	2.756	2.241	0.099	76.481
Housing services	14,737	2298	1363	126	21,720
Housing services/income	14,737	0.207	0.168	0.007	5.736
Total wealth	14,737	7483	23,384	−9945	387,018
Total wealth/house value	14,737	0.267	1.251	−2.006	62.871
Total wealth/income	14,737	0.405	1.383	−3.902	68.532
Annual nonhousing savings	254	391	7452	−32,801	59,891
Move indicator (to homeownership)	15,058	0.068	0.251	0	1
Move indicator (among rentals)	13,805	0.268	0.443	0	1
	Homeowners				
Income	30,796	27,568	16,571	2008	119,045
Family size	30,796	3.23	1.45	1	15
Age	30,796	44.78	12.42	25	70
House value	30,796	52,784	44,049	1543	472,339
House value/income	30,796	2.211	2.041	0.039	53.333
Housing services	30,796	3958	3303	116	35,425
Housing services/income	30,796	0.166	0.153	0.003	4.000
Annual mortgage payment	30,796	2227	2850	0	57,193
Mortgage indicator	30,796	0.733	0.442	0	1
Mortgage balance	30,796	19,565	23,092	0	280,271
Total wealth	30,796	69,038	80,406	−23,684	707,099
Nonhousing wealth	30,796	35,812	59,116	−9945	396,586
Home equity wealth	30,796	33,226	35,767	−15,728	436,264
Home equity/house value	30,796	0.632	0.319	−2	1
Total wealth/house value	30,796	1.450	2.288	−2.003	148.800
Total wealth/income	30,796	2.838	3.621	−1.358	84.152
Annual nonhousing savings	1854	63	11,828	−83,137	62,023
Move indicator (all moves)	30,207	0.068	0.245	0	1
Move indicator (to rental)	30,207	0.025	0.157	0	1
Move indicator (downgrade to smaller home)	30,207	0.014	0.117	0	1
Move indicator (upgrade to larger home)	30,207	0.029	0.169	0	1

NOTES: All dollar values are deflated by the nonhousing component of the consumer price index: base year 1980. House value for renters and housing services for owners are calculated using an owner's equivalent rent of 0.075. The conversions are calculated for illustrative purposes, but this particular value for owner's equivalent rent does not influence any parts of the estimation procedures. Annual payment is either rent for renters or mortgage payment for owners. Total wealth is the sum of home equity wealth and nonhousing wealth. Home equity wealth is zero for renters. Because we disregard moves by renters to another rental unit, we illustrate the move frequencies by excluding rent-to-rent moves. Moves (among rentals) is the frequency of rent-to-rent moves for the subsample of renters that do not switch to homeownership. There are some missing observations of the move indicator for a small fraction of the sample.

TABLE A.2
BIRTH COHORT SAMPLE MEANS

Birth cohort	1961–1965	1956–60	1951–55	1946–50	1941–45	1936–40	1931–35	1926–30	1921–25	1916–1920
Age	25	30	35	40	45	50	55	60	65	70
All Households										
Num. obs.	1971	6693	9073	7040	4362	3189	3556	3800	3237	1865
Income	20,036	21,314	23,138	27,807	28,709	26,881	25,041	23,329	17,490	14,113
House value (occupied)	36,841	39,571	43,668	54,861	51,076	48,334	50,272	46,984	40,012	36,954
House value (owned)	21,727	24,405	30,560	46,292	43,632	40,442	43,224	41,484	34,542	30,405
Total wealth/house value (occupied)	0.516	0.597	0.732	0.963	1.296	1.232	1.320	1.791	1.702	1.397
Total wealth	18,351	22,864	31,772	53,419	60,896	60,899	72,391	78,296	69,155	58,429
Home equity wealth	7505	9701	14,085	25,939	27,463	28,383	34,251	35,678	30,485	27,639
Nonhousing wealth	10,846	13,162	17,686	27,480	33,432	32,516	38,139	42,616	38,671	30,790
Renter indicator	0.468	0.473	0.406	0.260	0.241	0.258	0.241	0.191	0.229	0.329
Move indicator (ex. rent-to-rent)	0.111	0.097	0.086	0.082	0.068	0.045	0.038	0.039	0.033	0.028
Renters										
Num. obs.	963	3325	3783	1855	1058	831	860	721	736	612
Move indicator (to homeownership)	0.100	0.085	0.075	0.087	0.060	0.051	0.033	0.029	0.030	0.020
Homeowners										
Num. obs.	978	3285	5227	5150	3289	2350	2685	3076	2488	1248
Move indicator (all moves)	0.122	0.107	0.093	0.080	0.070	0.043	0.038	0.041	0.033	0.032
Move indicator (to rental)	0.048	0.038	0.035	0.029	0.024	0.019	0.016	0.014	0.010	0.011
Move indicator (downgrade)	0.024	0.018	0.013	0.012	0.017	0.011	0.010	0.013	0.015	0.012
Move indicator (upgrade)	0.050	0.051	0.045	0.039	0.029	0.013	0.013	0.014	0.008	0.009
Home equity ratio	0.347	0.411	0.460	0.550	0.639	0.696	0.775	0.835	0.879	0.900

NOTES: This table reports sample means for various birth cohorts. House value (occupied) is the reported house value for homeowners, and it is calculated from rental value using an owner's equivalent rent of 0.075 for renters. House value (owned) is the reported value for owners and is zero for renters. Because we disregard moves by renters to another rental unit, we illustrate the move frequencies by excluding rent-to-rent moves. See notes to Table A.1 for descriptions of other variables. The age heading is the approximate age of the households observed during the sample period 1980–1993. There is some sample truncation for the oldest and youngest cohorts because they were outside the 25–70-year age range.

TABLE A.3
HOUSING INVESTMENT POLICY FUNCTION OWNERS: ORDERED PROBIT

	Ordered Probit
Home size	−0.0025
	(0.0003)
Income	0.0105
	(0.0008)
Low wealth/house value ratio (<0.3)	0.8566
	(0.0781)
High wealth/house value ratio (>0.3)	0.0231
	(0.0020)
User cost	−0.0071
	(0.0029)
Family size	0.0592
	(0.0074)
Age	−0.0208
	(0.0071)
Age^2	0.0002
	(0.0001)
Rent/downgrade cut	−1.814
Downgrade/remain cut	−1.616
Remain/upgrade cut	2.104
Num obs	30,091
Switch to rent	760
Downgrade	415
Remain	28,031
Upgrade	885

NOTES: Ordered categories {rent, downgrade, remain in existing home, upgrade}. There is a linear spline in the wealth to house value ratio term with a knot at 0.3 to capture the effect of down-payment constraints. The reported coefficients are the marginal effects in each region of the spline. Standard errors are in parentheses. Sample is restricted to observations with nonmissing lagged observations. All dollar values are expressed in thousands of 1980 dollars.

TABLE A.4
HOUSING INVESTMENT POLICY FUNCTION RENTERS: PROBIT

	Probit
Income	0.0296
	(0.0013)
Low wealth/rental house value ratio (<0.1)	0.4845
	(0.2608)
Mid wealth/rental house value ratio (>0.1, <0.3)	1.544
	(0.2326)
High wealth/rental house value ratio (>0.3)	−0.0027
	(0.0103)
User cost	−0.0193
	(0.0044)
Family size	0.0322
	(0.0104)
Age	−0.0359
	(0.0118)
Age^2	0.0003
	(0.0001)
Constant	−1.131
	(0.2360)
Num. obs.	15,022
Remain renting	13,919
Switch to homeownership	1103

NOTES: Ordered categories {remain renting, switch to homeownership}. There is a linear spline in the wealth to rental house value ratio term with knot values at 0.1 and 0.3 to capture the effect of downpayment constraints. The rental house value is calculated from the rental value using an owner's equivalent rent ratio of 0.075. The reported coefficients are the marginal effects in each region of the spline. Standard errors are in parentheses. Sample is restricted to observations with nonmissing lagged observations. All dollar values are expressed in thousands of 1980 dollars.

TABLE A.5
UPGRADE/DOWNGRADE ADJUSTMENT SIZE POLICY FUNCTION FOR OWNERS

	Downgrades	Upgrades
Home size	−0.0028	−0.0087
	(0.0007)	(0.0006)
Income	0.0112	0.0048
	(0.0020)	(0.0012)
Low wealth/house value ratio (<.3)	−0.6779	0.4181
	(0.4311)	(0.2829)
High wealth/house value ratio (>.3)	−0.0105	0.0377
	(0.0120)	(0.0033)
User cost	−0.0014	−0.0024
	(0.0073)	(0.0046)
Family size	0.0418	0.0206
	(0.0204)	(0.0129)
Age	−0.0393	−0.0276
	(0.0176)	(0.0122)
Age^2	0.0004	0.0003
	(0.0002)	(0.0001)
Constant	0.4036	1.3258
	(0.3854)	(0.2622)
Num. moves	415	885
R^2	0.1218	0.3474

NOTES: Magnitude of adjustment size is measured as $|\log(h_{it}) - \log(h_{it-1})|$. The sample is restricted to households that move. There is a linear spline in the wealth to house value ratio term with a knot value at 0.3. The reported coefficients are the marginal effects in each region of the spline. Standard errors are in parentheses. Sample is restricted to observations with nonmissing lagged observations. All dollar values are expressed in thousands of 1980 dollars.

TABLE A.6
ADJUSTMENT SIZE POLICY FUNCTION FOR RENTERS

	Log House Value
Log income	0.7601
	(0.0345)
Wealth/income ratio	0.1070
	(0.0109)
User cost	−0.0048
	(0.00617)
Family size	−0.0151
	(0.0144)
Age	0.0332
	(0.0151)
Age^2	−0.0005
	(0.0002)
Constant	0.6318
	(0.3037)
Num. obs.	1019
R^2	0.3734

NOTES: Standard errors are in parentheses. Estimated on the subsample of households that switch from renting to homeownership. All dollar values are expressed in thousands of 1980 dollars.

TABLE A.7
RENTAL VALUE POLICY FUNCTION

	Log Rental Value
Log income	0.4598
	(0.0064)
Wealth/income ratio	0.0118
	(0.0033)
User cost	−0.0046
	(0.0020)
Home price index	0.0046
	(0.0011)
Family size	−0.0050
	(0.0027)
Age	0.0032
	(0.0029)
Age^2	−0.00006
	(0.00003)
Constant	−0.9591
	(0.1393)
Num. obs.	14,737
R^2	0.2990

NOTES: Standard errors are in parentheses. Estimated on the subsample of households that rent. Rent paid is missing for 1988 and 1989. All dollar values are expressed in thousands of 1980 dollars. Home price index is in real terms with the base year value set to 1.

TABLE A.8
ANNUAL SAVING POLICY FUNCTION

	Annual Saving
Income	0.2588
	(0.0194)
Total wealth	−0.0985
	(0.0040)
Rent/mortgage payment	−0.2518
	(0.1050)
Family size	−0.5120

(*Continued*)

TABLE A.8
CONTINUED.

	Annual Saving
	(0.1570)
Age	0.0676
	(0.1817)
Age^2	0.00007
	(0.00204)
Constant	−2.257
	(3.6804)
Num. obs.	2108
R^2	0.2304

NOTES: During our sample period, the PSID only collected wealth data in 1984 and 1989. We constructed the total contribution to wealth between 1984 and 1989 by taking the difference between wealth in 1989 and what the value of the household's 1984 wealth would have been in 1989 had it accumulated interest at the interest rates recorded in our data. We compute the annual saving amount using an arithmetic average: dividing the total contribution by 5 years. The income measure averages income across the five years between 1984 and 1989. Total wealth includes both nonhousing wealth and home equity wealth. Total wealth, rent, and annual mortgage payments are the values recorded in 1984. Renting or owning status is for the year 1984. All dollar values are expressed in thousands of 1980 dollars.

TABLE A.9
INCOME PROCESS

	$\log(y_{it})$
Age	0.1095
	(0.0022)
Age^2	−0.0011
	(0.00002)
Cohort	0.0114
	(0.0008)
Constant	−21.8481
	(1.5361)
ρ	0.4689
σ_ϵ	0.3085
σ_ν	0.8121
σ_a	0.6412
Observations	52,324
Households	7317
R^2	0.0965

NOTES: Random effects with AR(1) error term disturbance. Regressand: log income. Cohort is the head of household's birth year. Standard errors are in parentheses.

TABLE A.10
INTEREST RATE AND HOME INFLATION VAR

Interest Rate		Home Inflation	
r_{t-1}	0.615	π_{t-1}	0.534
	(0.105)		(0.147)
π_{t-1}	−0.435	r_{t-1}	0.113
	(0.086)		(0.178)
constant	2.845	*constant*	0.178
	(0.641)		(1.090)
Error covariance			
r	3.399		
π	2.543	9.823	
Num. years	33	1975–2009	

NOTES: Standard errors are in parentheses.

TABLE A.11
GOODNESS OF FIT: SIMULATED LIFE CYCLE OF YOUNGEST COHORT

Age	Initial(data)	30–35	35–40	40–45	45–50	50–55	55–60	60–65	65–70
		All Households							
Income	33,609	36,634	40,666	45,187	47,283	45,655	42,854	37,699	31,898
House value (occupied)	53,821	52,137	63,768	77,966	91,531	106,099	119,281	131,982	141,915
House value (owned)	35,477	41,479	56,061	72,330	85,960	100,721	113,112	125,180	134,760
Total wealth/house value (occupied)	0.610	0.908	1.164	1.499	1.737	1.828	1.853	1.819	1.811
Total wealth	34,022	46,665	67,055	100,034	131,864	158,160	176,976	186,689	187,681
Home equity wealth	13,510	13,749	19,131	25,973	34,045	44,320	55,682	68,087	79,933
Nonhousing wealth	20,511	32,915	47,924	74,061	97,819	113,840	121,285	118,602	107,748
Renter indicator	0.367	0.264	0.182	0.119	0.109	0.098	0.103	0.110	0.115
Move indicator (ex. rent-to-rent)	0.088	0.107	0.098	0.084	0.090	0.076	0.083	0.072	0.072
		Renters							
Move indicator (to homeownership)	0.110	0.201	0.208	0.192	0.213	0.178	0.184	0.137	0.122
		Homeowners							
Move indicator (all moves)	0.074	0.064	0.069	0.068	0.074	0.066	0.071	0.064	0.066
Move indicator (to rental)	0.033	0.022	0.024	0.021	0.024	0.019	0.020	0.019	0.018
Move indicator (downgrade)	0.006	0.011	0.013	0.010	0.014	0.009	0.015	0.012	0.013
Move indicator (upgrade)	0.035	0.031	0.033	0.037	0.036	0.037	0.036	0.033	0.036
Home equity ratio	0.372	0.363	0.362	0.376	0.413	0.448	0.493	0.534	0.570

This table reports life-cycle summary statistics generated from the forward simulations of the first stage policy functions for the five youngest cohorts in the sample. The simulations are conducted on the subsample of 450 households of family size 2 that we use for the second stage estimates. There are a total of 90 households in the youngest cohorts. The first column reports the statistics from the data for these households. The following columns report statistics in 5-year intervals. The age heading reports the age range of those households. See footnotes to Tables A.2 and A.1 for variable descriptions.

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